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**A New Extension Exponential Distribution with
Applications of COVID-19 Data**

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Abstract

In this paper, we introduce and study a new extension of exponential distribution with three-parameter named as the X-Gamma extension of the exponential (XGExEx) distribution. Survival function and hazard function for XGExEx distribution are discussed. Maximum likelihood estimation (MLE) and maximum product spacing (MPS) Methods for the XGExEx distribution parameters are discussed. A numerical study using real data analysis and Monte-Carlo simulation are performed to compare between different methods of estimation. Superiority of the new model over some well-known distributions are illustrated by COVID-19 Data. The XGExEx model can produce better fits than some well-known distributions as the Marshall–Olkin alpha power exponential (MOAPE) [Nassar et al. (2019)], extended odd Weibull exponential (EOWE) [Afify and Mohamed (2020)], Weibull-Exponential (WE) [Oguntunde et al. (2015)], ExEx and generalized exponential (GE) [Gupta and Kundu (2001)].

Keywords: X-Gamma; exponential distribution; maximum likelihood estimation; maximum product spacing; and COVID-19 data analysis.



الملخص باللغة العربية

امتداد جديد للتوزيع الأسّي مع التطبيق علي بيانات COVID-19

في هذا البحث ، قدمنا ودرسنا امتدادًا جديدًا للتوزيع الأسّي بثلاثة معلمات، ومن الممكن تسميته بامتداد X-Gamma extension of the exponential للتوزيع الأسّي X-Gamma extension of the exponential ويمكن ان يطلق عليه الاختصار التالي XGExEx. تمت مناقشة دالة البقاء ودالة الخطر لتوزيع XGExEx. وتمت مناقشة تقدير الإمكان الأعظم Maximum likelihood estimation (MLE) و (MPS) maximum product spacing لمعاملات توزيع XGExEx. ويتم إجراء دراسة عددية باستخدام تحليل البيانات الحقيقية ومحاكاة مونت كارلو للمقارنة بين طرق التقدير المختلفة. تم توضيح كفاءة النموذج الجديد بالمقارنة مع بعض التوزيعات المعروفة وذلك بالتطبيق على بيانات COVID-19. واتضح أن نموذج XGExEx أفضل ملاءمة من بعض التوزيعات المعروفة مثل توزيع Marshall-Olkin alpha power exponential وتوزيع extended odd Weibull وتوزيع Weibull exponential والتوزيع الأسّي exponential وتوزيع امتداد الأسّي extension of the exponential والتوزيع المعمم generalized exponential.

كذلك تم تقييم مرونة واعتمادية توزيع (XGExEx) المقترح من خلال مجموعتين من البيانات الحقيقية لفيروس كورونا المستجد، وذلك باستخدام بيانات يومية لمعدل الوفيات الخام لدولة المملكة المتحدة The United Kingdom وعددها 76 مشاهدة، وذلك من 15 ابريل الي 30 يونيو 2020. أما مجموعة البيانات الأخرى فهي بيانات يومية عن معدل الوفيات الخام لدولة اليابان Japan وعددها 38 مشاهدة، وذلك من 4 سبتمبر الي 12 أكتوبر 2020. وتم الحصول على هذه البيانات من إصدارات منظمة الصحة العالمية.

الكلمات المفتاحية: عائلة X-Gamma، توزيع extension of the exponential، طريقة الامكان الأعظم، maximum product spacing، تحليل البيانات

1. Introduction

The corona virus disease (COVID-19) is considered one of the worst pandemics in the recent years. Most countries have increased their health measures to decrease the spread rate of the COVID-19 by postponing education, closing borders and Applying partial curfew rules.

While Medical Researchers have spent lots of time to find medical solutions to return to normal life, other researchers have focused on the mathematical and statistical modeling of the COVID-19 outbreak to make future predictions. Remuzzi and Remuzzi (2020) predicted how many hospital beds and personnel Italy needed under the exponential trend. Maleki et al. (2020) forecasted the recovered and confirmed COVID-19 cases using autoregressive time series models based on the two-piece scale mixture normal distributions. Caccavo (2020) introduced the compartmental susceptible-infectious-recovered-deceased (SIRD) model differentiates between recovered and diseased cases within each compartment and predict COVID-19 cases in China and Italy. Ayyoubzadeh et al. (2020) predicted COVID-19 cases in Iran by using long –short term memory (LSTM) which is a deep-learning method. El-Morshedy et al. (2020) introduced a new discrete analogous of the generalized Lindley Distribution to model the daily new cases and deaths of COVID-19. Almetwally et al. (2020) introduced a new discrete analogous of the discrete Marshall–Olkin generalized exponential distribution to model the daily new cases and deaths of COVID-19 data. Almetwally and Gamal (2020) introduced a new discrete analogous discrete alpha power inverse Lomax distribution to analyze the daily count of new cases of the COVID-19 of Australia.

The extension of the exponential (ExEx) distribution was introduced by Nadarajah and Haghighi (2011). The cumulative distribution function (CDF) and the probability density function (PDF) of the ExEx distribution are respectively as follows

$$F(x; \varphi, \omega) = 1 - e^{[1-(1+\omega x)^\varphi]}; \quad x > 0, \varphi, \omega > 0, \quad (1.1)$$

$$f(x; \varphi, \omega) = \omega \varphi (1 + \omega x)^{\varphi-1} e^{[1-(1+\omega x)^\varphi]}. \quad (1.2)$$

Modified and extended versions of the ExEx distribution have been studied by many authors as Gómez et al. (2014) presented a new extension of the exponential distribution based on mixtures of positive distributions. Khan et al. (2015) studied transmuted generalized exponential distribution for analyzing lifetime data. Muhammad (2016) introduced a new family of distributions called the Poisson-odd generalized exponential distribution. De Andrade et al. (2016) introduced exponentiated generalized extended exponential



distribution. Kumar et al. (2017) discussed extended exponential distribution based on order statistics. Hassan et al. (2018) proposed a new distribution called the alpha power transformed extended exponential distribution. Pena-Ramirez et al. (2018) proposed a new lifetime model called the exponentiated power generalized Weibull distribution, which is obtained from the exponentiated family applied to the power generalized Weibull distribution.

During recent years, the X-Gamma (XG) distribution has been shown great interest by researchers, which is introduced by Sen et al. (2016). The quasi-XG distribution has been introduced by Sen and Chandra (2017). A new bounded distribution has been introduced by Altun and Hamedani (2018) using the transformation $Y = e^{-X}$ as an alternative to the beta distribution based on the XG distribution. Another generalization of XG distribution has been provided by Sen et al. (2018a) on the basis of a special mixture of exponential and gamma distributions. Parameter estimation of XG distribution under the progressively type-II censored sample has been studied by Sen et al. (2018b) by using different methods. The transmuted-XG distribution has been studied by Biçer (2019). Yadav et al. (2019) introduced inverse X-Gamma distribution using the transformation $Y = \frac{1}{X}$. Bantan et al. (2020) introduced the half-logistic XG distribution using half-logistic family. Sen et al. (2020) studied discrimination analysis between the Lindley and XG distribution.

On the other hand, the XG-Generator (XG-G) family has been proposed by Cordeiro et al. (2019) to incorporate any distribution into a larger family. The XG-G family has flexible shapes to model various lifetime data sets. The XG-G family add a parameter with one extra shape parameter $\vartheta > 0$, the CDF of XG-G family is given by

$$F(x; \vartheta, \psi) = 1 - \frac{[1 - G(x; \psi)]^\vartheta}{\vartheta + 1} \left\{ 1 + \vartheta - \vartheta \ln(1 - G(x; \psi)) + 0.5\vartheta^2 [\ln(1 - G(x; \psi))]^2 \right\}, \quad (1.3)$$

where $G(x; \psi)$ is a baseline cdf with a parameter vector ψ . The PDF of XG-G family can be expressed as

$$f(x; \vartheta, \psi) = \frac{\vartheta}{\vartheta + 1} g(x; \psi) [1 - G(x; \psi)]^{\vartheta-1} \left\{ \vartheta + 0.5\vartheta^2 [\ln(1 - G(x; \psi))]^2 \right\} \quad (1.4)$$

where $g(x; \psi) = dG(x; \psi)/dx$.

The aim of this paper is to clarify two things. Firstly, propose and study a new lifetime distribution called X-Gamma extension of the exponential (XGExEx) distribution based on the XG-G family. Some statistical properties of XGExEx distribution are provided. Secondly,

parameters estimation for the XGExEx distribution are discussed by using MLE and MPS methods. To evaluate the performance of the estimators, extensive simulation study is carried out. Our XGExEx model as well as some other well-known distributions are illustrated by COVID-19 Data. The XGExEx distribution can produce better fits than these other distributions.

The paper is organized as follows; in section 2, we introduce the description, notation and the statistical properties of XGExEx distribution. In section 3 we discuss the parameter estimation of XGExEx distribution. In section 4, Monte-Carlo simulation study are presented to compare the performance of the parameter's estimation for different methods. In section 5, application of COVID-19 Data is studied. Lastly in section 6, we discuss the results and conclusions of the current study.

2. Model Description and Notation

In this section, we will introduce the XGEXE distribution and some of its sub-models.

2.1.XGExEx Distribution

The XG-G family and ExEx distribution have been used to generate XGExEx distribution. It is represented by the random variable $X \sim XGExEx(\vartheta, \varphi, \omega)$. By using Equations (1.3, 1.4, 1.1 and 1.2), the CDF of XGExEx distribution takes this form

$$F(x; \vartheta, \varphi, \omega) = 1 - \frac{e^{\vartheta[1-(1+\omega x)^\varphi]}}{\vartheta + 1} \{1 + (1 + \omega x)^\varphi(\vartheta - \vartheta^2) + 0.5\vartheta^2[1 + (1 + \omega x)^{2\varphi}]\}, \quad (2.1)$$

where $\vartheta, \varphi, \omega > 0$ and $x > 0$. The PDF of XGL distribution is given as:

$$f(x, \vartheta, \varphi, \omega) = \frac{\vartheta}{\vartheta + 1} \omega \varphi (1 + \omega x)^{\varphi-1} e^{\vartheta[1-(1+\omega x)^\varphi]} \{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x)^\varphi]^2\} \quad (2.2)$$

Figure 1 display plots of the PDF of the XGExEx distribution for some parameters values as follows

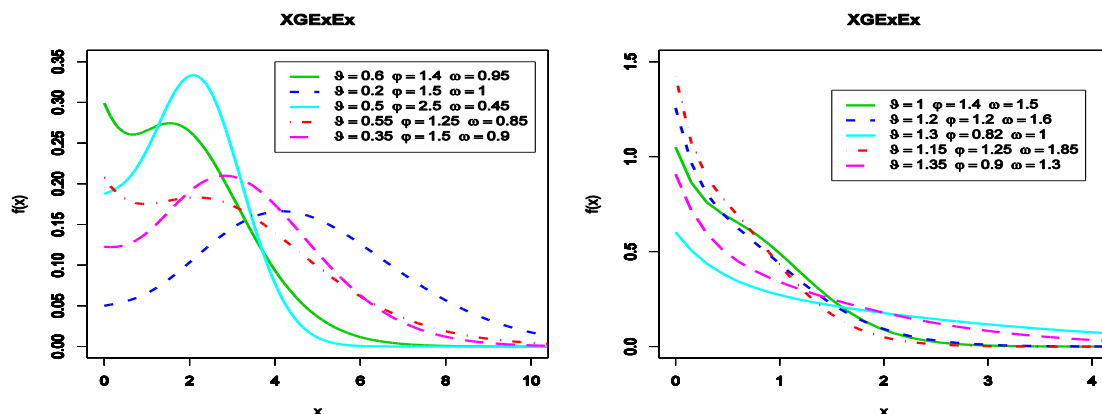


Figure 1. Plots of the PDF of the XGExEx distribution with Some Values of Parameters.



Therefore, a random variable with PDF (2.2) is denoted by $X \sim \text{XGExEx}(\vartheta, \varphi, \omega)$ reduces to the two parameters, this is a new model can be denoted as X-Gamma exponential (XGEx) distribution when $\varphi \rightarrow 1$.

2.2. Reliability functions of XGExEx distribution

The survival function of XGExEx distribution is given by

$$S(x; \vartheta, \varphi, \omega) = \frac{e^{\vartheta[1-(1+\omega x)^\varphi]}}{\vartheta + 1} \{1 + (1 + \omega x)^\varphi(\vartheta - \vartheta^2) + 0.5\vartheta^2[1 + (1 + \omega x)^{2\varphi}]\} \quad (2.3)$$

The hazard function of a lifetime random variable X with XGExEx distribution is given by

$$h(x; \vartheta, \varphi, \omega) = \frac{\vartheta\omega\varphi(1 + \omega x)^{\varphi-1} \{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x)^\varphi]^2\}}{\{1 + (1 + \omega x)^\varphi(\vartheta - \vartheta^2) + 0.5\vartheta^2[1 + (1 + \omega x)^{2\varphi}]\}} \quad (2.4)$$

Figure 2 display plots of the hazard function of the XGExEx distribution for some values of the parameters as follows

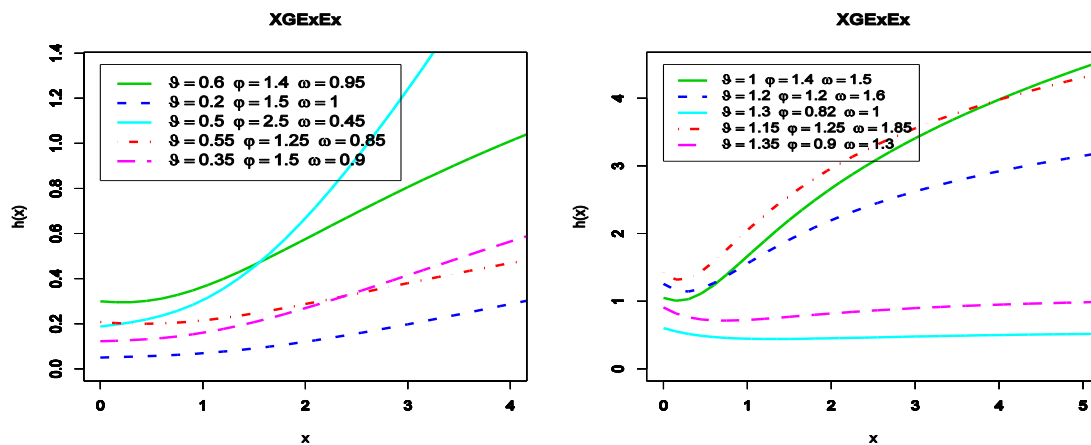


Figure 2. Plots of the hazard of the XGExEx with Some Values of the Parameters.

3. Parameter Estimation

In this section, the parameter estimation for the XGExEx distribution using MLE and MPS methods will be discussed in details.

3.1. MLE method

The log-likelihood function of XGExEx distribution, is given by:

$$l(\vartheta, \varphi, \omega) = n \ln\left(\frac{\vartheta}{\vartheta + 1}\right) + n[\ln(\varphi) + \ln(\omega)] + n\vartheta - \sum_{i=1}^n (1 + \omega x_i)^\varphi + (\varphi - 1) \sum_{i=1}^n \ln(1 + \omega x_i) + \sum_{i=1}^n \ln\{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x_i)^\varphi]^2\}. \quad (3.1)$$

Equation (3.1) can be maximized directly by using the R package with an optimization function, to solve the non-linear likelihood equations

obtained by differentiating Equation (3.1) with respect to $\vartheta, \varphi, \omega$ and equating to zero. The non-linear likelihood equations are given as

$$\frac{\partial l(\vartheta, \varphi, \omega)}{\partial \vartheta} = \frac{n}{\vartheta(\vartheta + 1)} + n + \sum_{i=1}^n \frac{1 + \vartheta[1 - (1 + \omega x_i)^\varphi]^2}{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x_i)^\varphi]^2},$$

$$\frac{\partial l(\vartheta, \varphi, \omega)}{\partial \varphi} = \frac{n}{\varphi} - \sum_{i=1}^n (1 + \omega x_i)^\varphi \ln(1 + \omega x_i) + \sum_{i=1}^n \ln(1 + \omega x_i)$$

$$+ \sum_{i=1}^n \frac{\vartheta^2[1 - (1 + \omega x_i)^\varphi] \ln(1 + \omega x_i)}{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x_i)^\varphi]^2},$$

and

$$\frac{\partial l(\vartheta, \varphi, \omega)}{\partial \omega} = \frac{n}{\omega} - \varphi \sum_{i=1}^n x_i(1 + \omega x_i)^{\varphi-1} + (\varphi - 1) \sum_{i=1}^n \frac{x_i}{1 + \omega x_i}$$

$$+ \vartheta^2 \varphi \sum_{i=1}^n x_i \frac{[1 - (1 + \omega x_i)^\varphi](1 + \omega x_i)^{\varphi-1}}{\vartheta + 0.5\vartheta^2[1 - (1 + \omega x_i)^\varphi]^2}.$$

3.2.MPS Method

MPS method is used to estimate the parameters of continuous univariate models as an alternative to the MLE method. The uniform spacings of a random sample $x_1 < \dots < x_n$ of size n from the XGExEx distribution can be defined by

$$D_i(\vartheta, \varphi, \omega) = F(x_i; \vartheta, \varphi, \omega) - F(x_{i-1}; \vartheta, \varphi, \omega); i = 1, 2, \dots, n + 1$$

where D_i denotes to the uniform spacings and $\sum_{i=1}^{n+1} D_i = 1$. The MPS estimators can be obtained by maximizing

$$G(\vartheta, \varphi, \omega) = \frac{1}{n + 1} \sum_{i=1}^{n+1} \ln(D_i(\vartheta, \varphi, \omega))$$

To more information of MPS method, see Cheng and Amin (1983) and Almetwally and Almongy (2019_{b, a}).

The natural logarithm of the product spacing function of XGExEx distribution is given by

$$\ln G(\vartheta, \varphi, \omega) = \frac{1}{n + 1} ([\ln(1 - \mathcal{F}(x_1; \vartheta, \varphi, \omega)) - \ln(\mathcal{F}(x_n; \vartheta, \varphi, \omega))] + \ln(\mathcal{F}(x_{i-1}; \vartheta, \varphi, \omega) - \mathcal{F}(x_i; \vartheta, \varphi, \omega))) \tag{3.2}$$

Where $\mathcal{F}(x_i, \vartheta, \varphi, \omega) = \frac{e^{\vartheta[1-(1+\omega x_i)^\varphi]}}{\vartheta+1} \{1 + (1 + \omega x_i)^\varphi(\vartheta - \vartheta^2) + 0.5\vartheta^2[1 + (1 + \omega x_i)^{2\varphi}]\}$.

The partial derivatives of MPS with respect to the unknown parameters cannot be solved explicitly, so numerical methods like the conjugate gradients algorithms can be used to calculate the MPS of $\vartheta, \varphi, \omega$.



4. Simulation Study

In this section; a Monte Carlo simulation is done to estimate the parameters by using MLE and MPS methods. R packages were used and the Simulation algorithm was as follows:

Monte Carlo experiments were carried out based on 1000 random sample for following data generated form XGExEx distribution by using numerical analysis in R packages, where x_i is distributed as XGExEx distribution for different parameters $\Omega = (\vartheta, \varphi, \omega)$ with different actual values of parameter and for different samples sizes $n = 35, 75$ and 150 . We can find the parameter estimation by using Equations (3.1 and 3.2) and an optim function in R packages. We could define the best method as which minimizes the Bias and mean squared error (MSE) of the estimator. From Tables (1-2) we can see that all the estimates reveal the property of consistency, the Bias and MSE decrease when n increase. Also, can see that the MPS method has more relative efficiency than MLE.

Table 1: MLE and MPS estimation methods for XGExEx distribution with different values of parameters when $\vartheta = 0.5$

$\vartheta = 0.5$		φ	0.5				1.4				2.2			
ω	n		MLE		MPS		MLE		MPS		MLE		MPS	
			Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
0.5	35	ϑ	-0.0007	0.1385	0.0425	0.0218	-0.0389	0.2070	-0.0100	0.0315	-0.0330	0.1778	0.0326	0.0385
		φ	0.0227	0.0786	-0.0194	0.0052	0.1461	0.2963	-0.0216	0.0853	0.1016	0.2790	-0.0493	0.0421
		ω	-0.0131	0.0906	0.0421	0.0093	0.0561	0.2759	0.1506	0.1189	0.0584	0.2410	0.0356	0.0314
	75	ϑ	0.0044	0.1275	0.0420	0.0208	-0.0052	0.1139	0.0136	0.0141	0.0005	0.1320	0.0391	0.0208
		φ	0.0107	0.0529	-0.0142	0.0027	0.0394	0.1841	-0.0441	0.0294	0.0177	0.1314	-0.0412	0.0180
		ω	-0.0409	0.0815	0.0144	0.0088	0.0139	0.1375	0.0544	0.0322	0.0160	0.0950	0.0033	0.0110
	150	ϑ	0.0032	0.0758	0.0144	0.0054	-0.0147	0.1033	-0.0117	0.0114	-0.0063	0.0875	0.0045	0.0082
		φ	0.0068	0.0360	-0.0067	0.0012	0.0141	0.1437	-0.0432	0.0240	-0.0090	0.1257	-0.0387	0.0145
		ω	-0.0118	0.0633	0.0107	0.0038	0.0116	0.1057	0.0410	0.0257	0.0280	0.0910	0.0257	0.0107
1.4	35	ϑ	-0.0058	0.1452	0.0568	0.0282	-0.0144	0.1422	0.0191	0.0619	-0.0823	0.1132	-0.0425	0.0047
		φ	0.0192	0.0731	-0.0196	0.0050	0.1169	0.2848	-0.0574	0.0906	0.4312	0.7019	0.0564	0.0312
		ω	-0.0018	0.0481	0.0227	0.0037	-0.0335	0.3389	0.0584	0.0900	-0.0049	0.4563	0.0563	0.0158
	75	ϑ	0.0103	0.1306	0.0400	0.0193	0.0158	0.1406	0.0315	0.0202	0.0076	0.1031	-0.0302	0.0037
		φ	0.0092	0.0504	-0.0132	0.0026	0.0654	0.2036	-0.0299	0.0333	0.0824	0.2920	-0.0483	0.0893
		ω	-0.0342	0.0443	0.0129	0.0507	-0.0417	0.3233	0.0619	0.1163	-0.0027	0.3049	0.0544	0.1258
	150	ϑ	-0.0013	0.0738	0.0179	0.0059	0.0012	0.0823	0.0182	0.0075	0.0028	0.0861	0.0214	0.0078
		φ	0.0051	0.0347	-0.0075	0.0012	0.0257	0.1288	-0.0218	0.0143	0.0188	0.1428	-0.0408	0.0248
		ω	-0.0036	0.0403	0.0090	0.0011	-0.0101	0.1849	0.0204	0.0287	0.0025	0.1398	0.0157	0.0253
2.2	35	ϑ	-0.0090	0.1432	0.0559	0.0278	-0.0101	0.1617	0.0315	0.0244	-0.0170	0.1386	0.0732	0.1433
		φ	0.0201	0.0723	-0.0187	0.0048	0.1400	0.2510	-0.0249	0.0336	0.0643	0.2738	-0.1272	0.3056
		ω	-0.0017	0.0403	0.0140	0.0040	-0.1369	0.4310	0.0343	0.0693	0.0476	0.3037	0.0332	0.1891
	75	ϑ	0.0040	0.1178	0.0366	0.0166	0.0060	0.1182	0.0339	0.0156	0.0068	0.1310	0.0356	0.0191
		φ	0.0088	0.0503	-0.0133	0.0026	0.0396	0.1533	-0.0368	0.0225	0.0583	0.2430	-0.0427	0.0854
		ω	-0.0258	0.0393	0.0252	0.0819	-0.0340	0.3068	0.0532	0.1133	-0.0061	0.2326	0.0490	0.1268
	150	ϑ	-0.0019	0.0730	0.0191	0.0061	0.0013	0.0760	0.0208	0.0068	0.0031	0.0865	0.0208	0.0078
		φ	0.0049	0.0346	-0.0075	0.0012	0.0119	0.1003	-0.0270	0.0097	0.0354	0.1622	-0.0288	0.0269
		ω	0.0008	0.0373	0.0043	0.0019	-0.0016	0.1458	0.0169	0.0169	-0.0133	0.1613	0.0050	0.0257



Table 2: MLE and MPS estimation methods for XGEx distribution with different values of parameters when $\vartheta = 1.4$

$\vartheta = 1.4$		φ	0.5				1.4				2.2			
ω	n		MLE		MPS		MLE		MPS		MLE		MPS	
			Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
0.5	35	ϑ	0.1116	0.5067	0.1764	0.3832	-0.0749	0.4133	-0.0030	0.1973	-0.0543	0.4567	0.1313	0.2781
		φ	0.0477	0.1445	-0.0287	0.0135	0.0889	0.3480	-0.1625	0.1379	0.0229	0.2918	-0.1864	0.1713
		ω	0.0208	0.3303	0.1535	0.1599	0.0824	0.2909	0.2405	0.1906	0.1006	0.3209	-0.0536	0.0737
	75	ϑ	0.0577	0.2965	0.0781	0.0906	-0.0276	0.3469	-0.0063	0.1241	0.0288	0.4677	0.2344	0.2898
		φ	0.0210	0.0875	-0.0183	0.0060	0.0634	0.3157	-0.1182	0.0940	0.0803	0.2743	-0.0772	0.0730
		ω	-0.0032	0.1959	0.0603	0.0436	0.0703	0.2531	0.1704	0.1214	-0.0182	0.2737	0.1094	0.0936
	150	ϑ	-0.0220	0.2386	0.0038	0.0815	-0.0438	0.2576	-0.0522	0.0740	0.0257	0.3064	-0.0015	0.0903
		φ	0.0124	0.0592	-0.0108	0.0033	0.0069	0.1983	-0.1051	0.0519	0.0716	0.1949	0.0751	0.0635
		ω	0.0049	0.1823	0.0591	0.0426	0.0654	0.2290	0.1507	0.0886	0.0289	0.3679	0.0910	0.1495
1.4	35	ϑ	-0.0132	0.4109	0.1664	0.3288	0.0694	0.5642	0.1770	0.4271	0.0091	0.5103	0.2151	0.4327
		φ	0.0413	0.1278	-0.0282	0.0136	0.2184	0.4854	-0.0934	0.1637	0.1726	0.5564	-0.1837	0.3096
		ω	0.0336	0.4305	0.1745	0.2905	-0.0225	0.4855	0.2360	0.3525	0.0512	0.4297	0.1135	0.1739
	75	ϑ	0.0398	0.3314	0.1154	0.1275	0.0135	0.3414	0.0802	0.1320	0.0290	0.3814	0.1246	0.2134
		φ	0.0171	0.0834	-0.0210	0.0063	0.0665	0.2630	-0.0831	0.0714	0.1034	0.4280	-0.1663	0.2034
		ω	-0.0313	0.3294	0.0483	0.1094	-0.0014	0.2256	0.1098	0.0752	0.0139	0.3839	0.1397	0.1981
	150	ϑ	-0.0168	0.1751	0.0371	0.0405	0.0357	0.3043	-0.0098	0.1034	-0.0017	0.2533	0.0636	0.0771
		φ	0.0108	0.0560	-0.0115	0.0031	0.1084	0.2173	-0.0555	0.0742	0.0331	0.2446	-0.0904	0.0579
		ω	0.0153	0.1920	0.0530	0.0459	0.0034	0.2048	0.0925	0.0703	0.0164	0.1721	0.0446	0.0283
2.2	35	ϑ	-0.0348	0.8984	0.1773	0.3371	0.0843	0.5793	0.1869	0.4680	0.0354	0.5607	0.2522	0.4787
		φ	0.0397	0.1275	-0.0289	0.0138	0.2332	0.4953	-0.0914	0.1709	0.2000	0.6273	-0.1701	0.2971
		ω	0.1350	0.9074	0.1636	0.2684	-0.0454	0.5100	0.2529	0.4015	-0.0350	0.4366	0.0967	0.1765
	75	ϑ	0.0356	0.3245	0.1164	0.1372	0.0289	0.3679	0.0910	0.1495	0.0579	0.4205	0.1341	0.2126
		φ	0.0169	0.0829	-0.0213	0.0064	0.0803	0.2743	-0.0772	0.0730	0.1787	0.4864	-0.1209	0.2092
		ω	-0.0315	0.3334	0.0549	0.1294	-0.0182	0.2737	0.1094	0.0936	-0.0380	0.3585	0.0856	0.1423
	150	ϑ	-0.0158	0.1774	0.0385	0.0386	0.0257	0.3064	-0.0015	0.0903	0.0129	0.2767	0.0656	0.0810
		φ	0.0107	0.0557	-0.0115	0.0030	0.0751	0.2532	-0.0536	0.0704	0.0643	0.2470	-0.0787	0.0762
		ω	0.0113	0.1664	0.0485	0.0368	0.0129	0.2468	0.0923	0.0829	-0.0242	0.2871	0.0692	0.0969

5. Application of Real Data Analysis

This section is devoted to illustrate the potentiality of the XGExEx distribution for Two real data sets. XGExEx distribution is compared with other competitive models, namely: the Marshall–Olkin alpha power exponential (MOAPE) [Nassar et al. (2019)], extended odd Weibull exponential (EOWE) [Afify and Mohamed (2020)], Weibull-Exponential (WE) [Oguntunde et al. (2015)], ExEx and generalized exponential (GE) [Gupta and Kundu (2001)].

Tables 3, 4 provide values of Cram´er-von Mises (W^*), Anderson-Darling (A^*) and Kolmogorov- Smirnov (KS) statistic along with its P-value for the all models fitted based on Two real data sets. In addition, these tables contain the MLE and standard errors (SE) of the parameters for the considered models.

The first data represents a COVID-19 data belong to The United Kingdom of 76 days, from 15 April to 30 June 2020. these data formed of drought mortality rate. The data are as follows: 0.0587 0.0863 0.1165 0.1247 0.1277 0.1303 0.1652 0.2079 0.2395 0.2751 0.2845 0.2992 0.3188 0.3317 0.3446 0.3553 0.3622 0.3926 0.3926 0.4110 0.4633 0.4690 0.4954 0.5139 0.5696 0.5837 0.6197 0.6365 0.7096 0.7193 0.7444 0.8590 1.0438 1.0602 1.1305 1.1468 1.1533 1.2260 1.2707 1.3423 1.4149 1.5709 1.6017 1.6083 1.6324 1.6998 1.8164 1.8392 1.8721 1.9844 2.1360 2.3987 2.4153 2.5225 2.7087 2.7946 3.3609 3.3715 3.7840 3.9042 4.1969 4.3451 4.4627 4.6477 5.3664 5.4500 5.7522 6.4241 7.0657 7.4456 8.2307 9.6315 10.1870 11.1429 11.2019 11.4584.

Table 3. MLE, SE, KS test and P-values for COVID-19 data of The United Kingdom

		estimate	SE	KS	P-Value	W^*	A^*
XGExEx	ϑ	2.7766	2.1801	0.0604	0.9282	0.0727	0.5083
	φ	0.5275	0.1458				
	ω	0.6279	0.6680				
ExEx	φ	0.5839	0.1113	0.0617	0.9172	0.0744	0.5186
	ω	1.1204	0.4511				
GE	β	0.8009	0.1171	0.0985	0.4245	0.1395	0.9287
	λ	0.3530	0.0569				
WE	α	14.5701	16.5203	0.0806	0.6769	0.1196	0.8048
	β	0.8252	0.0751				
	λ	0.0169	0.0207				
EOWE	α	1.9148	0.6136	0.0672	0.8594	0.0751	0.5531
	β	8.8195	5.8230				
	λ	1.5439	0.6527				
MOAPE	α	0.0021	0.0012	0.0615	0.9189	0.0736	0.5421
	β	0.0322	0.0160				
	λ	0.2553	0.1400				

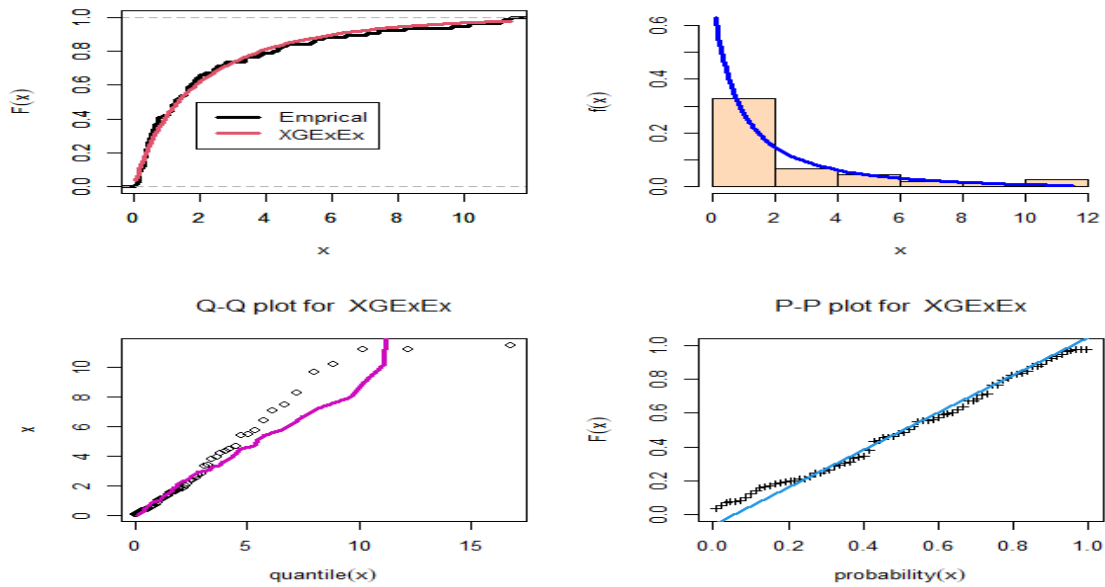


Figure 3. Cumulative function and empirical cdf, histogram and the Fitted XGExEx distribution, Q-Q plot and P-P plot for the XGExEx distribution for COVID-19 data of The United Kingdom.

The second data represents a COVID-19 data belong to Japan of 38 days, from 4 September to 12 October 2020. these data formed of drought mortality rate. The data are as follows: 0.1596 0.2733 0.1142 0.0851 0.1976 0.2243 0.1810 0.0828 0.1504 0.2169 0.0404 0.1208 0.1334 0.1589 0.1184 0.1698 0.0648 0.1027 0.0511 0.1019 0.1520 0.1006 0.0624 0.0372 0.1112 0.0859 0.0854 0.0847 0.1443 0.0836 0.0238 0.0355 0.0353 0.0937 0.0349 0.0924 0.0344 0.0228.

Table 4. MLE, SE, KS test and P-values for COVID-19 data of Japan

		estimate	SE	KS	P-Value	W*	A*
XGExEx	ϑ	0.2339	0.0234	0.0805	0.9495	0.0415	0.3079
	φ	1.4372	0.0027				
	ω	41.7987	0.3739				
ExEx	φ	27.1401	NA	0.1961	0.0936	0.0460	0.3280
	ω	0.2290	NA				
GE	β	3.2913	0.8781	0.1347	0.4558	0.0695	0.4558
	λ	17.7892	2.9010				
WE	α	3.6476	7.1388	0.0930	0.8671	0.0461	0.3294
	β	1.5658	0.3052				
	λ	2.9314	2.6984				
EOWE	α	1.6494	0.3822	0.0956	0.8454	0.0433	0.3174
	β	0.4200	0.5177				
	λ	6.3243	1.1032				
MOAPE	α	25.0328	60.7498	0.0972	0.8314	0.0441	0.3324
	β	22.8634	5.5231				
	λ	2.3743	3.1073				

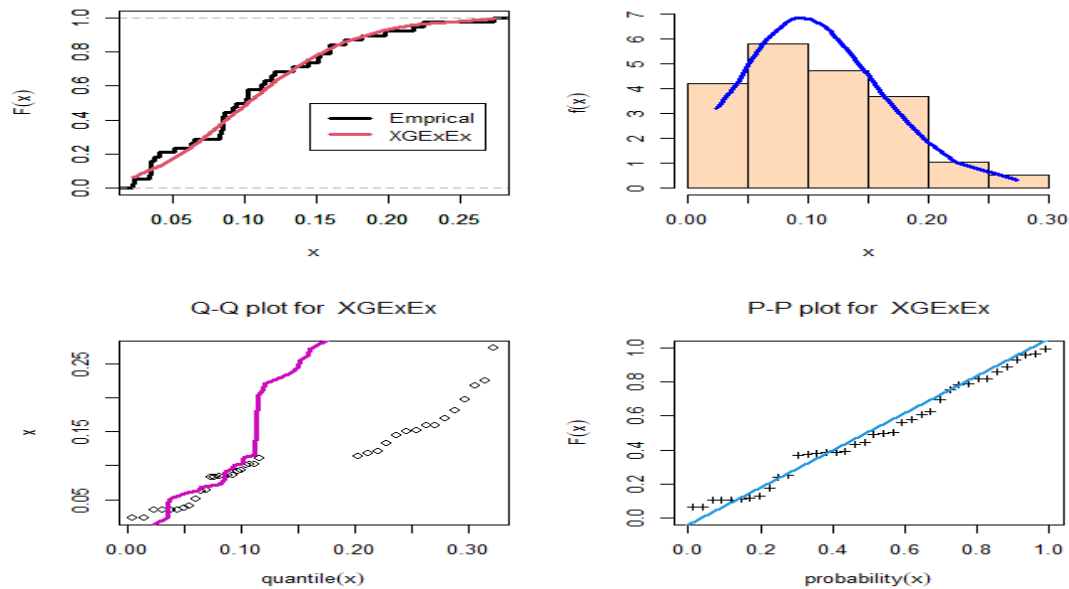


Figure 4. Cumulative function and empirical cdf, histogram and the Fitted XGExEx distribution, Q-Q plot and P-P plot for the XGExEx distribution for COVID-19 data of Japan.

In tables 3, 4, the XGExEx model has the highest p-value and the lowest distance (D) of Kolmogorov-Smirnov(K-S), W^* and A^* value when it compares with all other models that used here to fit the COVID-19 data. Figure 3,4 shows the fit empirical, histogram, QQ-plot and PP-plot for the XGExEx distribution for COVID-19 data of the United Kingdom and Japan.

6. Conclusion

In this paper, we propose a new three-parameter distribution, called the X-Gamma extension of the exponential distribution which can be denoted as XGExEx distribution. The importance of the XGExEx distribution is derived from the wide utilization of the exponential model in life testing and the flexibility to analyze lifetime data. Survival and hazard functions of the XGExEx distribution are introduced. We provide some applications of XGExEx distribution in the context of statistics. The parameter estimation of XGExEx distribution is derived by MLE and MPS. The methods of estimation are employed to estimate the model parameters and simulation results are provided to evaluate the model performance. Two real-life data proposed model provides a consistently better fit than the ExEx, GE, WE, EOWE, MOAPE, and exponential distributions. Finally, we hope that the proposed model will be of great use in the application in various fields.



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