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Bent-Cable Regression of Normally Distributed Data Applied to the Fifth Covid-19 Wave in Egypt

by

Samar Ahmed Helmy

Lecturer, Department of Statistics, Mathematics& Insurance

Faculty of commerce- Port Said University

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Abstract:

The bent-cable model is used to describe the fifth Covid-19 wave in Egypt The data are normally distributed but rightly skewed The model comprises two linear segments, joined smoothly by a quadratic bend. The class of bent cables includes, as a limiting case, the popular piecewise-linear model (with a sharp kink), otherwise known as the broken stick. Associated with bent-cable regression is the estimation of the bend width parameter, through which the abruptness of the underlying transition may be assessed. We present worked the recorded fifth Covid-19 Wave in Egypt to demonstrate the regularity and irregularity of bent-cable regression encountered in finite-sample settings. We also presented a new method to determine the two inflection points by drawing a line from the CTP to the lowest point in each side of the curve, the points of intersection between the drawn lines and the real data will be the points of inflection. The validity of the suggested method is validated. Under such conditions, the least-squares estimators are shown (i) to be consistent, and (ii) to asymptotically follow a multivariate normal distribution. Hence the least-squares estimators are used to analysis the considered data.

***Keywords*: fifth wave - Covid-19 - normal distribution - bent-cable - critical time point - transition points**

1 - Introduction

A regression framework to model a change point data with transition between two segments was introduced by Chiu et al (2006), they assumed a quadratic bend to model the transition zone between the two linear segments, they called this frame work; bent cable model.

A major difficulty in estimating bent cable model is estimating the transition zone.

The bent-cable model may lead to an interval [x_1, x_2] which may be wider or narrower than what could possibly be necessary to suitably describe the transition zone. So, accuracy of the bent-cable estimates depends on the precision of the transition period.

In this paper a tackle to this problem in a systematic procedure. If the variable is normal, and symmetric, then we use the properties of this distribution to calculate the inflection points.

The proposed method is applied to analyze the 5th wave of covid-19 which exhibit a normal distribution. In the following section a brief review is presented while the proposed method is presented in section 3 and the last section is the applied study followed by a discussion.

2 - Preview

The review will include both methodology review and applied study review.

- ### Methodology review:

Bacon and Watts (1971) introduced a smooth change point model by taking into account a different parameterization of picewise linear model. They replaced the sign function by a transition function which must satisfy some conditions to insure a gradual transition if the parameter has a large value and has an abrupt transition if the parameter's value is close to zero but the interpretation parameters depend on the shape of the transition. a consequence of this Hout (2010) stated that inference about the slopes of the two linear phases is not possible.

To overcome this problem Chiu et al. (2006) introduced bent cable regression model. In this model the transition phase is considered as a quadratic function. Inference about the slopes of the two linear phases is possible. Khan et al. (2009) proposed Bayesian method to inference the slopes of the linear incoming and linear outgoing segments.

Khan and Kar (2018) stated that in practice , bent cable model may led to a wider or narrower interval than it should be to describe the transition zone.

- **The applied study review:**

Al-Hussein and Tahir (2020) used generalized model SEIR (Susceptible, Exposed, Infectious and Recovered) individuals to simulate the ongoing spread of the disease and forecast the future behavior of the outbreak. The authors of Amar et. al.(2020) applied seven regression analysis-based models to analyze the database of COVID-19 for Egypt from 15 Feb to 15 Jun 2020, and predicted with the number of patients that will be infected with COVID-19, and estimated the epidemic final size.

Boldog et al. (2020) developed a computational tool to assess the risks of novel Covid-19 outbreaks outside China. They estimated (1) the evolution of the cumulative number of cases in China outside the closed areas, (2) connectivity of the destination country with China, including the effect of travel restrictions, baseline travel frequencies, and the efficacy of entry screening at destination, and (3) the efficacy of control measures in the destination country

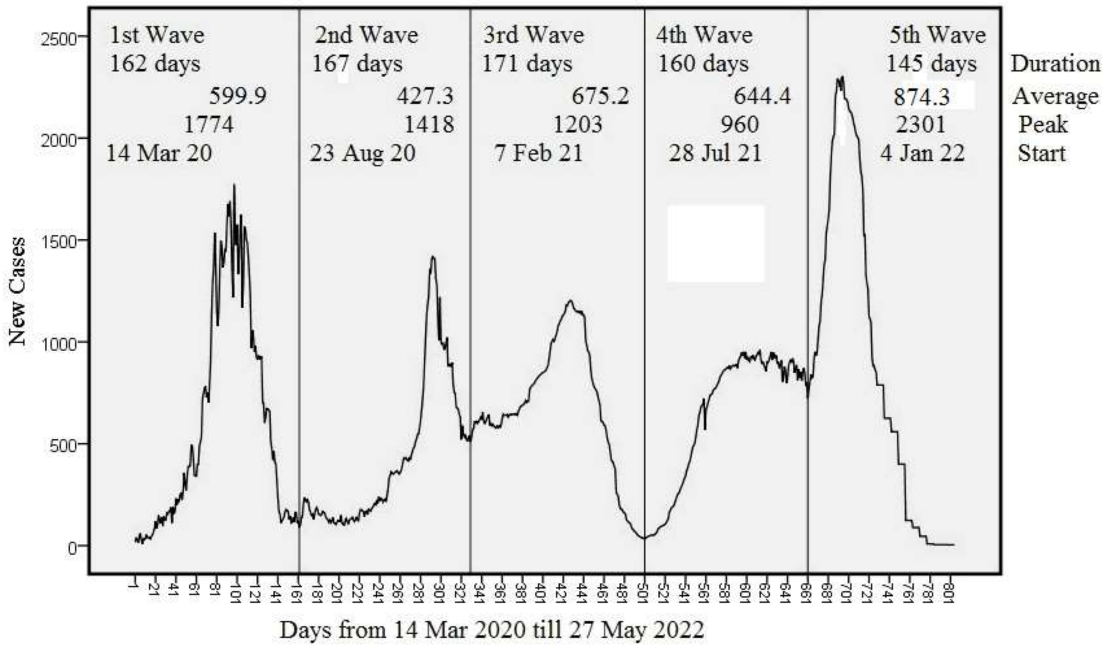
Helmy (2020) built and econometric model to estimate the impact of lockdowns to mitigate the effects of corona on the Egyptian GDP through the period April-June 2020. Helmy (2021), Statistically analysis of Covid-19 data of Egypt from 14 Mar 2020 - 3 Nov 2021 and forecasted the fourth corona wave.

Kuniya.(2020) analyzed the real-time data from 15 January to 29 February 2020 to predict the epidemic peak for COVID-19 in Japan using SEIR model. Using a least-square-based method with Poisson noise, he estimated the basic reproduction number for the pandemic is 2.6 (95 %). The epidemic peak could possibly reach the early-middle summer. Schüttler et al (2020) built a Gauss Model based on data from 2 Apr 2020 as a simple, analytically tractable model to make predictions on the corona pandemic from 25 countries during corona pandemic first wave and study the model's predictions. They provide evidence that this simple model might still have predictive power and was used to forecast the further course of the fatalities caused by Covid-19 per country, including peak number of daily deaths, date of peak, and duration of most deaths occur.

3 - Covid-19 in Egypt

As mentioned above, corona pandemic in Egypt started on 14 Mar 2020. Five waves may be distinguished. By a wave we mean the distance between two successive bottoms. The following figure shows these waves

Figure (1)
The five Covid-19 waves of new cases in Egypt



The first wave started 14 Mar 2020 with one case reached its peak on the 98th day reaching 1774 new case on 19 Jun 2020 decreased to 89 cases as its bottom, it lasted for 162 days.

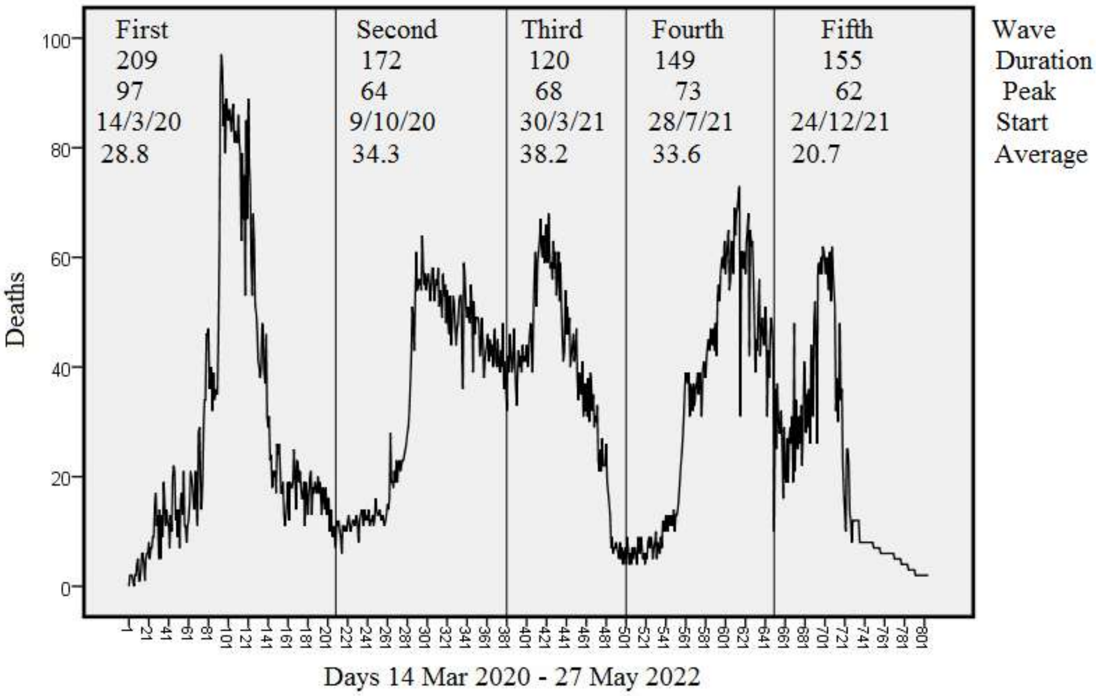
The second wave started 23 Aug 2020 with 103 new case reached its peak on day 293 reaching 1418 new case on 31 Dec 2020 decreased to 509 cases as its bottom, it lasted for 167 days.

The third wave started 7 Feb 2021 with 534 case reached its peak on day 428 reaching 1203 new case on 15 May 2021 decreased to 37 cases as its bottom, it lasted for 171 days.

The fourth wave started 28 Jul 2021 with 38 case reached its peak on day 614 reaching 960 new case on 17 Oct 2021 decreased to 723 cases as its bottom, it lasted for 160 days.

The fifth wave started 18 Oct 2021 with 769 case reached its peak on day 695 reaching 2301 new case on 6 Feb 2022 decreased to 4 cases weekly average of the week 21 - 27 May 2022 as its bottom, it lasted for 145 days. The Egyptian ministry of health stopped publishing data on new cases, deaths and recovered as fropm28 May 2022.

Figure (2)
The five Covid-19 waves of deaths in Egypt



Deaths also followed the same pattern as the new cases, were we had five waves as shown above. Death ratio varied through the five showing the effect of vaccination

Table (1)
Death ratio of the five waves

Wave	First	Second	Third	Fourth	Fifth
Total deaths	6018	5896	4587	5006	3215
Total new cases	97178	71360	115457	103097	126775
Ratio %	6.19	8.26	3.97	4.86	2.54

It is clear from the table above that the death ratio decreased in the last three waves. Since the vaccination started 4 Mar 2021, its effect appeared as from the third wave which started 30 Mar 2021.

5 - International Comparison

New cases / population ratio of Egypt is 0.49% putting Egypt in the 186th position out of 230 country and area compared to the same ratio of the world 6.87% . Thus Egypt is considered a very moderate case. . The deaths / new cases ratio of Egypt is 4.77% almost four folds the world ratio 1.16%, which may considered a serious case.

Table (2)

New cases ratio and death ratio

New cases / Population			Deaths / New cases		
Order	Country	Ratio %	Order	Country	Ratio %
1	Faeroe Islands	70.40	1	MS Zaandam	22.22
2	Gibraltar	56.46	2	Yemen	18.18
3	Andorra	56.06	3	Western Sahara	10.00
4	Iceland	55.29	4	Sudan	7.92
5	Denmark	51.43	5	Peru	5.93
8	Portugal	49.93	9	Egypt	4.77
30	Luxembourg	38.99	10	Afghanistan	4.24
47	Australia	30.12	70	Cameroon	1.61
48	Italy	29.68	71	Congo	1.60
58	USA	26.30	83	Morocco	1.36
59	Norway	26.17	85	Libya	1.28
92	Brazil	14.73	93	India	1.21
99	Qatar	13.42	97	USA	1.18
103	Russia	12.60	98	Saudi Arabia	1.17
128	Japan	7.29	99	World	1.16
133	World	6.87	104	Venezuela	1.09
140	Iraq	5.55	165	Germany	0.52
163	Saudi Arabia	2.19	171	Portugal	0.47
184	Algeria	0.59	177	Austria	0.43
185	Ghana	0.51	179	Finland	0.42
186	Egypt	0.49	193	UAE	0.25
193	Togo	0.43	199	Denmark	0.21
218	Faeroe Islands	0.08	205	Taiwan	0.16
222	Niger	0.03	213	Singapore	0.10
228	Macao	0.01	221	Cook Islands	0.02

It is horrible when more than 70% of the population are corona-virus infected as in Faeroe Islands, however its death is 0.08%. is relatively low but still is tedious to medically care for this part of the community.

4 - Methodology

As shown in the preview, most authors used SIR and SIER models to analyze Covid-19 data in Egypt or other countries. In this study bent-cable regression is used, may be for the first time, to analyze the covid-19 data in Egypt. Bent-cable analysis fits for some natural phenomena that call for models which exhibit a structural change, sometimes in the form of a difference in slopes. In such cases, the cause and onset of the change are often of major interest.

The bent-cable regression function, evaluated at the covariate value *x*, is

$f(x; \alpha, \beta_1, \beta_2, \mu, \theta) = \alpha + \beta_1 x + \beta_2 g(x; \mu, \theta)$, **where**

$$g(x;\mu,\theta)=\frac{(x-\mu+\theta)^2}{4\theta} I\{|x-\mu|\le\theta\}+(x-\mu)I\{x-\mu>\theta\} \qquad (1)$$

The observations $\{(x_i, Y_i)\}$ $i=1,\ldots, n$ generated by a basic bent-cable function are considered The regression model is

$$Y_i = g(x_i ; \lambda_0) + \varepsilon_i \text{, where } \varepsilon_i \text{ are iid.} \sim N(0, \sigma^2) \text{ for a known } \sigma^2 \qquad (2)$$

And *g* is the basic bent-cable, $\lambda_0 = (x_0, \theta_0)$ is the underlying bent-cable parameter. For the estimation procedure, we consider the unbounded parameter space, $\Omega = (-\infty, M] \times [0, \infty)$ for λ_0, and the open regression domain, $X = \mathbb{R}$. Here, M is some large positive but finite upper bound for the candidate *x*-values. The natural lower bound for candidate θ-values is, of course, zero. Any basic bent-cable $g(x; \lambda)$ for $\lambda \in \Omega$ is a candidate model in estimating λ_0. While searching only within the class of basic bent-cables (as opposed to the class of all three-stage models in Feder 1975, we do allow more flexibility than does Gallant (1974) and 1975, or Ivanov (1997).

Where

I(A) is an indicator function that equals 1 if A is true and 0 otherwise;

α and β_1 are the intercept and slope of the linear incoming stage, respectively;

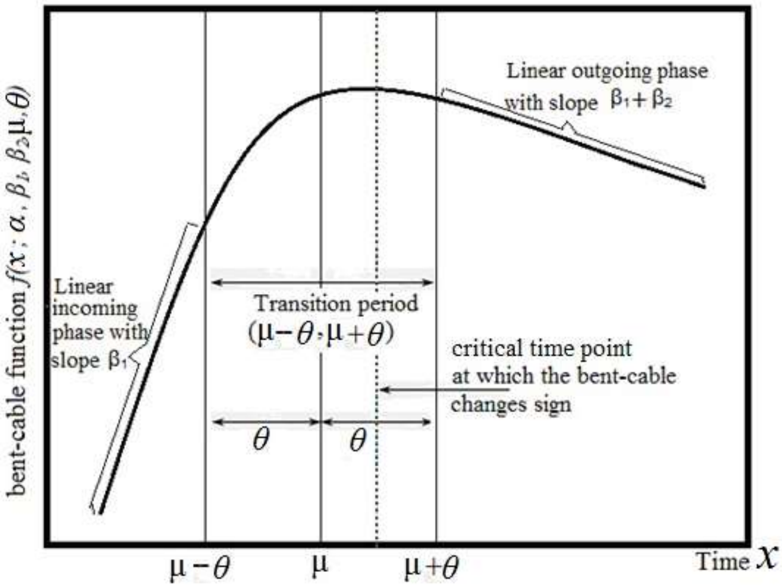
- **$\beta_1+\beta_2$ is the slope of the linear outgoing stage;**
- **μ and θ are the transition parameters, where μ, and [$\mu - \theta$, $\mu + \theta$] are the center and width of the bend, respectively; and**
- **ε_i is the random error component.**

Under this formulation, the transition begins at time $x_1 = x - \theta$ and ends at $x_2 = x + \theta$, and the CTP (critical time point) at which the slope of the bent-cable changes sign is $x-\theta-\frac{2\beta_1\theta}{\beta_2}$ (Chiu and Lockhart 2010).

Figure (3)

Graphical presentation of the bent-cable function

$f(x; \alpha, \beta_1, \beta_2, \mu, \theta) = \alpha + \beta_1 x + \beta_2 g(x; \mu, \theta)$



The bent-cable model assumes a quadratic bend (Equation (1)) to describe the transition zone. In application, the quadratic function of the bent-cable model may lead to an interval [x_1, x_2] which may be wider or narrower than what could possibly be necessary to suitably describe the transition zone.

If the data proof to be normally distributed this will be used in two directions:

1 - Calculating the critical time point, which is top of the normal curve corresponding to the mean μ of the distribution.

2 - Detecting the inflection points which are at μ- δ and μ+ δ which are the limits of the transition period

On the other hand results of normal estimates are compared to those of the bent-cable.

5 - Analysis of the fifth corona wave

Considering the 805 daily datum on corona composing five waves. Statistically, the five waves differ from each other. Testing the five waves for normality we get:

Figure (4-a)
Normality test for wave (1)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Wave one is normal with mean 603.04 and standard deviation 538.36.	One-Sample Kolmogorov-Smirnov Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure (4-b)
Normality test for wave (2)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Wave two is normal with mean 425.79 and standard deviation 359.16.	One-Sample Kolmogorov-Smirnov Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure (4-c)
Normality test for wave (3)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Wave three is normal with mean 672.39 and standard deviation 325.63.	One-Sample Kolmogorov-Smirnov Test	.002	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure (4-d)
Normality test for wave (4)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Wave four is normal with mean 648.68 and standard deviation 318.15.	One-Sample Kolmogorov-Smirnov Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure (4-e)

Normality test for wave (5)

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of X is normal with mean 1,085.41 and standard deviation 752.15.	One-Sample Kolmogorov-Smirnov Test	.084	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figures (4-a) ... (4-d) show that the first four waves are not normally approximated, while figure (4-e) show acceptance of hypothesis that the fifth wave is normally approximated with mean 1085.41 and standard deviation 751.46 at a significance level 5% and right skewed (skewness = 0.232)

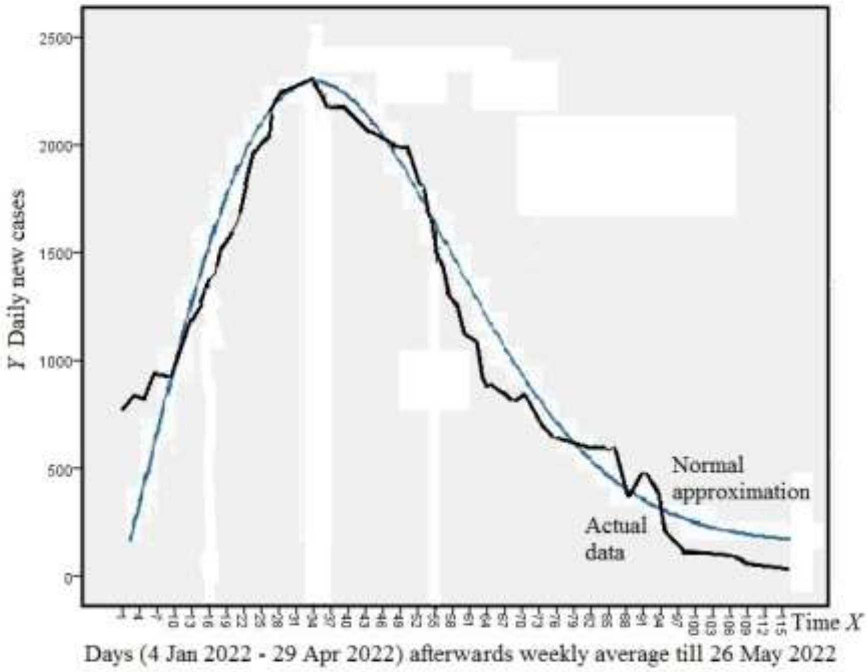
The following figure (5) shows the normal distribution with the above parameters:

Descriptive Statistics

N	Mean	Std. Deviation	Skewness	Kurtosis
116	1085.41	752.146	0.232	– 1.288

Figure (5)

Wave (5) ~ N(1085.41, 752.146, skewed 0.232, kurtosis - 1.288)



5. Bent-Cable regression

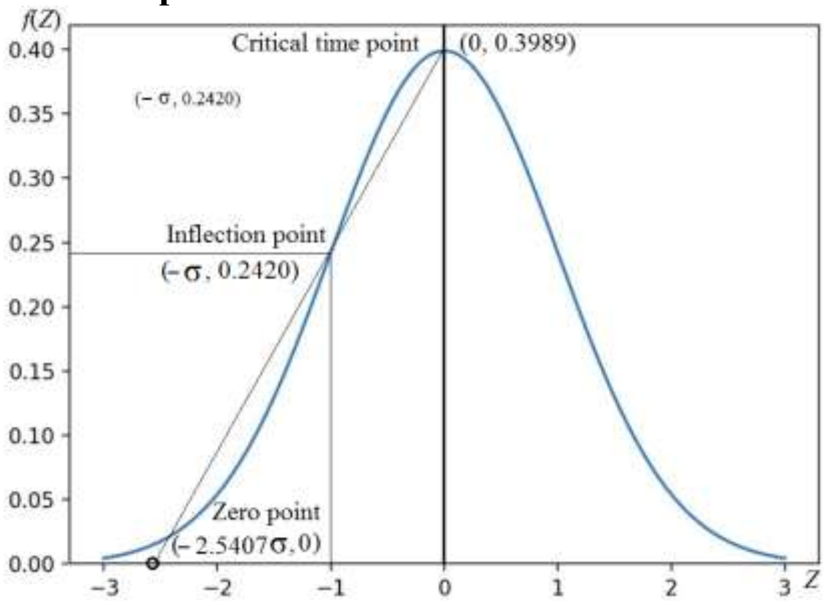
A suggested method to determine the transition period

Since the present distribution is not symmetric then we follow another procedure.

Consider the standard normal distribution shown in figure (6). The inflection points are at (μ ± σ), since the distribution is symmetric, we deal with left hand side and apply the results to the other side. If we draw a line from the critical time point which corresponds to the mean = 0 i.e the point (0 , 0.3989 σ) and the point (- 2.5407 σ , 0) it intersects with curve at the inflection point as shown in figure (6).

The suggested method is to make use of this fact that the inflection points of the normal distribution are at μ ± σ. Hence either we normalize the data using $z = \frac{x-\mu}{\sigma}$ **or use μ instead of zero and σ instead of one.**

Figure (6)
Inflection points of the standard normal distribution



This method cannot be applied to the case studied since its distribution is not asymmetric. Making use of the same approach, since the data are normally skewed distribution, the two inflection points may be determined as follows:

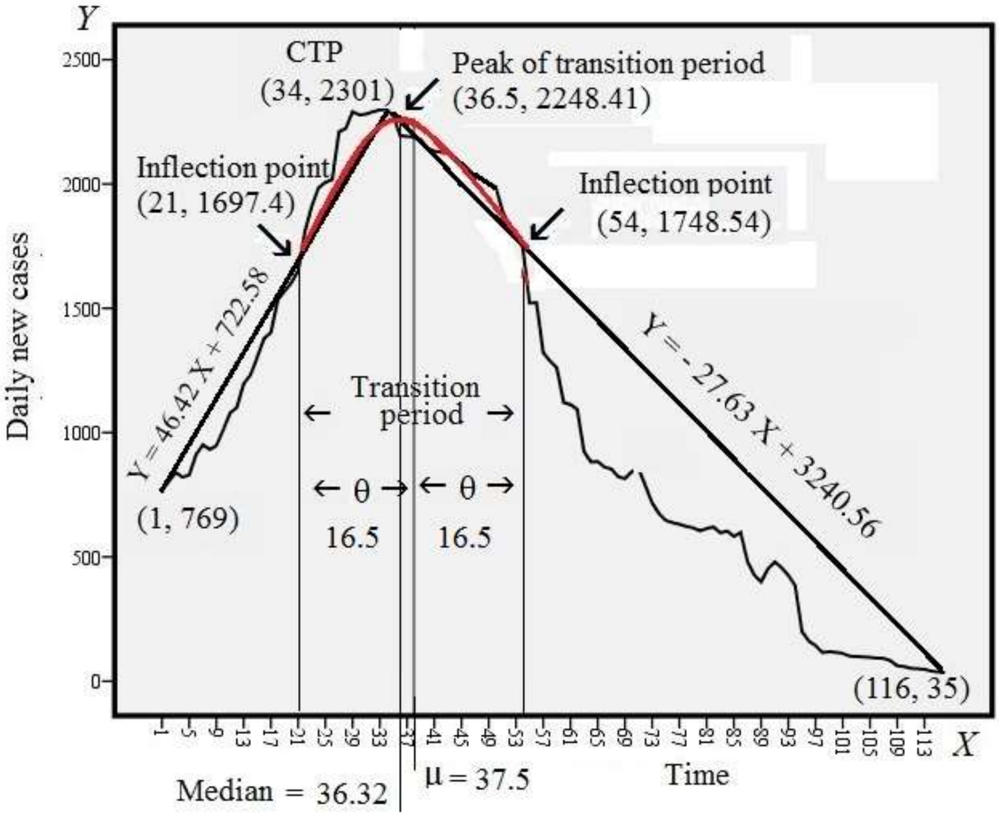
1 - Draw a line from the CTP to the lowest point in each side of the curve.

2 - The points of intersection between the drawn lines and the real data will be the points of inflection.

The procedure is illustrated in the following figure (7)

Figure (7)

Determination of the points of inflection and the transition period



From the two points (34, 2301) and (1, 769) calculate the equation of line which will be $Y = 46.42\ X + 722.58$ on the left hand side it intersects with curve of the actual data at the point (21, 1697.4) which is the left inflection point. Similarly, equation of the line on the right hand side is will be $Y = -\ 27.63\ X + 3240.56$ that intersects with curve of the actual data at the point (54, 1748.54) which is the right inflection point

Using SPSS-20 and the data between the two points, i.e between day 21 (25 Jan 2022). and day 54 (27 Feb 2022) The quadratic equation of the bent part of the model is estimated as:

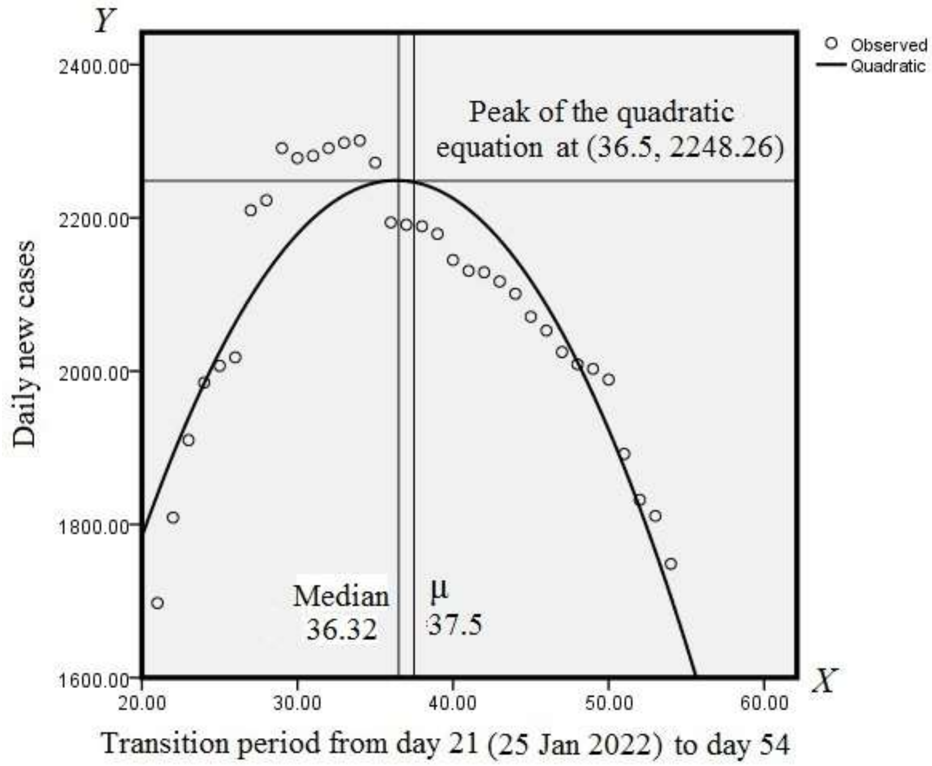
$$Y = -\ 50.251 + 126.556\ X - 1.742\ X^2 \qquad (3)$$

The following figure (8) illustrates this equation. It is worth noting that the peak of this curve occurs at the median (36.32) and not the mean (37.5) which is expected since the data follow a right skewed normal distribution as shown above. If we differentiate equation (3) with respect to *X* we get the slope equation:

$$\frac{dY}{dX} = 126.556 - 3.484\,X \qquad \textbf{(4)}$$

equate with zero so the maximum (or minimum) occurs at *X* = 36.32 which is the median. The second derivative of equation (3) $\frac{d^2Y}{dX^2} = -3.484$ **which means that at *X* = 36.32 of the quadratic equation is maximum.**

figure (8)
The quadratic equation of the model



Length of the transition period is the distance between the two inflection points 54 - 21 = 33 = 2 θ. $\Rightarrow$ **θ = 16.5**

Now we have all parameters of equation (1) : μ = 37.5 and θ = 16.5 by substitution we get estimated equation :

$$g(x, 37.5, 16.5) = \frac{(x-54)^2}{66} I\{|x-37.5| \le 16.5\} + (x-37.5)\, I\{x-37.5 > 16.5\} \qquad \textbf{(5)}$$

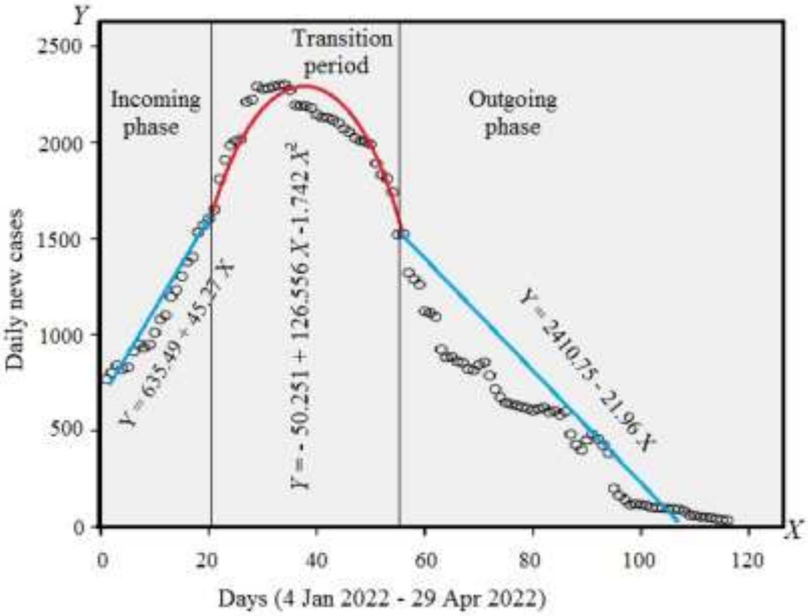
Using the data between day 1 and day 20, the linear incoming equation is estimated :

$$Y = 45.27\,X + 635.49 \qquad (6)$$

Using the data between day 54 and day 116, the linear outgoing equation is estimated :

$$Y = -\,27.63\,X + 3240.56 \qquad (7)$$

figure (9)
Three phases of the model



6. Validity of the suggested method

To determine the validity of the suggested method, we take two narrower points and two wider points and compare the estimates of the threes methods to decide which is the best.

Let the narrower points be (24 , 1985) and (50 , 1989), the quadratic equation will be :

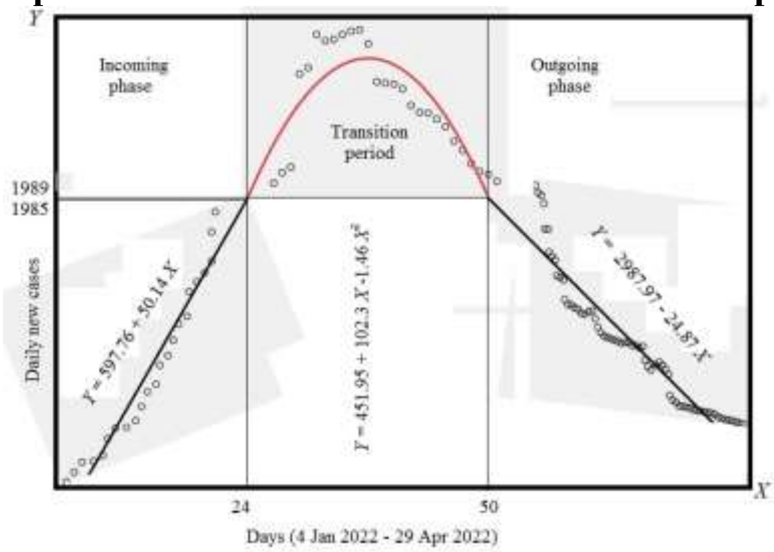
$$Y = 451.95 + 102.3\,X - 1.46\,X^2 \qquad (8)$$

and the two adjacent lines are:

incoming : $Y = 597.76 + 50.14\,X \qquad (9)$

outgoing : $Y = 2987.97 - 24.87\,X \qquad (10)$

figure (10)
Three phases of the model with narrower inflection points



Let the wider points be (17 , 1403) and (55 , 1521), the quadratic equation will be :

$$Y = -505.03 + 150.8\,X - 2.05\,X^2 \qquad (8)$$

and the two adjacent lines are:

$$\text{incoming} : \quad Y = 687.33 + 38.64\,X \qquad (9)$$

$$\text{outgoing} : \quad Y = 2360.7 - 21.44\,X \qquad (10)$$

Ffigure (11)
Three phases of the model with wider inflection points

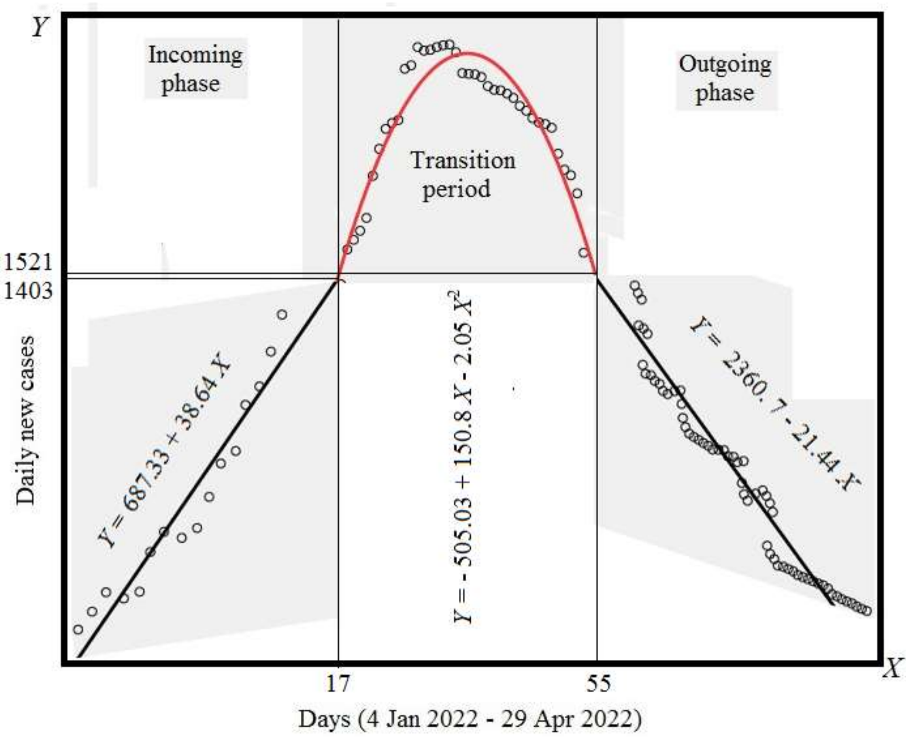


Table (3)
Comparison among some estimates of the two models

Day	Actual (X)	Narrow (N)	X - N	Suggested (S)	X - S	Wide (W)	X - W
1	769	647.9	121.1	680.76	88.24	725.97	43.03
10	1011	1099.16	- 88.16 -	1088.19	-77.19-	1073.73	- 62.73 -
20	1603	1600.56	3.56	1540.89	62.11	1690.97	87.97
30	2278	2206.95	71.05	2178.63	99.37	2173.97	104.03
40	2145	2207.95	- 62.95 -	2064.79	80.21	2246.97	-101.97-
50	1989	1916.95	72.05	1922.55	66.45	1909.97	79.03
60	1121	1495.77	- 374.77 -	1093.15	27.85	1074.3	46.7
70	843	1247.07	- 404.07 -	873.55	-30.55-	859.9	- 16.9 -
80	605	998.37	- 393.37 -	653.95	-48.95-	645.5	- 40.5 -
90	452	749.67	- 297.67 -	434.35	17.65	430.4	21.6
100	116	500.94	- 384.94 -	214.75	-98.75-	216.7	- 100.7 -
110	59	252.27	- 193.27 -	-4.85	63.85	2.3	56.7

6 - Conclusion

Table (3) shows that the estimate of the suggested model are much better than that of narrower or wider models. The absolute difference between the actual data and each of the three models of all cases shown in the next
table (4) confirms the above conclusion

Table (4)
Absolute differences between actual data and each of the three models

Model	N	Minimum	Maximum	Sum	Mean	Std. Deviation
Narrower	116	.30	514.29	23676.46	204.1074	161.28627
Suggested	116	1.13	342.01	9005.17	77.6308	59.18343
Wider	116	3.71	9551.97	324798.24	2799.9848	2831.94822
Valid N (listwise)	116					

The comparison based on the absolute difference since negative difference is so important as the positive one. The suggested method has less total difference than the other two models. Also the differences are more stable having a Std. Deviation of 59.2 compared with that of the other two models which are 161.3 and 2831.9 respectively.

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