مجلة البحوث المالية والتجارية
المجلد (23) – العدد الرابع – أكتوبر 2022

Using Johnson Schumacher Model for Parameter Estimation of Nonlinear Regression Model

Maie M. Kamel
Faculty of Commerce, Tanta University, Egypt
maie.kamel@commerce.tanta.edu.eg

Mona Nazih Abdel Bary
Faculty of Commerce, Suez Canal University, Egypt
mona_abdelbary@commerce.suez.edu.eg

Hanaa Abd El-Reheem Salem
Faculty of Commerce, Tanta University, Egypt
Hanaa_salem@commerce.tanta.edu.eg

رابط المجلة: https://jsst.journals.ekb.eg/

Abstract

In this study, we aim to estimate parameters of nonlinear model by using ordinary least square. This paper used a real data about Egypt cover the period from 2000 to 2017. The data set on exchange rate, inflation, exports, imports, investments, and budget deficit. The appropriate models of the data are cubic and Johnson Schumacher Model. The results of application data appear that the Johnson Schumacher nonlinear regression model is outstanding performance.

Keywords: Nonlinear Regression, Cubic Model, Johnson Schumacher Model, Nonlinear Least Square, Maximum Likelihood.

1. Introduction:

The model of nonlinear regression is a method of finding the relationship between the dependent variable and independent variables. Unlike traditional model of linear regression, which restricted to estimating linear models. Nonlinear regression can be estimated with arbitrary relationships between independent and dependent variables. This is accomplished using iterative algorithms of estimation. This approach is not necessary for simple polynomial model of the form$y = A + BX^2$. By defining $W = X^2$, we get a simple linear model, $Y = A + BW$ which can be estimated using traditional methods like the Linear Regression approach. Many models that appear nonlinear at first can be transform to a linear model, which can be analyzed using the model of linear regression approach.

In function forms in regression, an equation is linear in the variables if plotting the function of X any Y generates straight line:

$Y = \beta_0 + \beta_1 X + u$ **Linear in variable**

$Y = \beta_0 + \beta_1 X^2 + u$ **Linear in coefficients**

$Y = \beta_0 + \beta_1 X^{\beta_1} + u$ **Not linear in coefficients**

OLS method is restricted to models that are linear in the parameters

$Y = \beta_0 + \beta_1 X^2 + u$ **Can be estimated by OLS**

$Y = \beta_0 + \beta_1 X^{\beta_1} + u$ **Can not be estimated by OLS models that are nonlinear in parameters can estimate using nonlinear least squares. An iterative procedure which searches for the parameter value(s) which minimize the residual sum squares (RSS) of the model.**

This paper is organized as follows: Literature review is given in Section (2). Methodology is given in Section (3). The empirical study is introduced in Section (4). A Conclusion and Remark are given in

Section (5).

2. Literature Review

Bijan Payandehl (1983) presented several nonlinear models with forestry application. Motulsky, M.J et al (2006) described method for identifying outliers when fitting data with model of nonlinear regression. Chen (2010) investigated the performance for model of nonlinear and nonparametric regression with real data set simulated under a model of nonlinear. Xiao et al (2011) demonstrated that the error distribution determined which method performed better, with nonlinear regression better characterizing data with additive, comparison of multiple linear and nonlinear regression, autoregressive integrated moving average, artificial neural network, and wavelet artificial neural network methods. Adamowski et al (2012) indicated that coupled wavelet neural network models were a potentially promising method of urban water demand forecasting.

Kaneko and Funatsu (2013) proposed predictive performance criteria for models of nonlinear regression without cross validation. Tang and Zhao (2013) proposed empirical likelihood methods to estimate unknown regression parameters in β and the response mean θ in model of nonlinear regression with missing responses. S.V. Archontoulis et al (2014) indicated that the choice of the main function was critical distinguished nonlinear models from linear models had difficult without guidance. Limaa et al (2015) tested extreme learning machine for nonlinear regression and presented four nonlinear prediction methods. Excluding large datasets, extreme learning machine tends to be the fastest among the nonlinear models and the major result was that no single method was best for all the datasets. Huang et al (2016) introduced the large desperation for the estimation of least squares in nonlinear model of regression. Soner and Hasan (2016) introduced develop the reliable alternative approach of parameter estimation based on the particle swarm optimization algorithm in nonlinear regression model. Khan et al (2017) presented high breakdown and efficient estimation procedure for model of nonlinear regression used michaelis-menten model and gompertz model. Mahaboob et al. (2017) indicated that generally optimal estimators for the parameters of nonlinear model. Tian and Hao (2018) indicated that the combined method can sufficiently incorporate the advantages of individual models. However, the applying of linear combination is limited so the possibility of nonlinear terms ignored. Waki (2018) proposed a branch and bound

search algorithm for a mixed integer nonlinear programming formulation of the Akaike's information criterion minimization, David et al. (2019) estimated generalized least square method for estimating the parameters of the nonlinear split plot design models. This study introduces nonlinear models and focus on two nonlinear regression models; cubic and Johnson Schumacher. We aim to estimate parameters of the nonlinear models applying on exchange rate and comparing between the nonlinear models to select the appropriate models for the real data set.

3. Methodology

3.1 Johnson Schumacher Model:

We use the Johnson Schumacher model introduced by Johnson (1935) and Schumacher (1939). The model of parameters estimated from the observed data. The parameters in the fitted equations are used to assess the importance in the variables. In this study, we focus on the following Johnson Schumacher model;

$$y = \beta_0 exp\left(-\beta_1 (x + \beta_2)\right) + \varepsilon, \quad (1)$$

Where parameters are; both $\beta_0, \beta_1, and\ \beta_2 > 0$, y is the dependent variable and x is independent variable, which is a special case of regression model;

$$y = g\left(x_1, x_2, \dots, x_p | \beta\right) + \varepsilon, \quad (2)$$

3.2 Estimation Methods:

In this section, we can introduce methods of estimation parameters. Non Linear Least squares and Maximum Likelihood are proposed.

3.2.1 Nonlinear Least Squares Estimators:

The development of the least of squares estimations for the nonlinear model gives about complications not encountered in the linear model. This is simply illustrated in the case of the exponential regression model of Equation (3):

$$y = \alpha\, e^{\beta x} + \varepsilon \quad (3)$$

Given a set of data (y_i, x_i) for $i = 1, 2, \dots, n$, the estimate of α and β find using minimizing

$$SS_{Res} = \sum_{i=1}^{n}\left(y_i - \alpha e^{\beta x_i}\right)^2 \quad (4)$$

We differentiate the result of Equation (4) with respect to α and β and equal set of each derivative to zero. These yield the following equations:

$$\sum_{i=1}^{n}\left(y_i - \widehat{\alpha}e^{\widehat{\beta}x_i}\right)\left(-e^{\widehat{\beta}x_i}\right) = 0 \qquad (5)$$

$$\sum_{i=1}^{n}\left(y_i - \widehat{\alpha}e^{\widehat{\beta}x_i}\right)\left(-\widehat{\alpha}e^{\widehat{\beta}x_i}.\ x_i\right) = 0 \qquad (6)$$

Equation (5) and (6) are nonlinear in the parameter estimators $\widehat{\alpha}$, $\widehat{\beta}$. Thus we cannot compute estimates by elementary matrix algebra. Some type of iterative process must be used.

Properties of the least squares estimators. Consider as a general formulation, the model

$$y_i = f(x_i, \theta) + \varepsilon_i, \qquad i = 1, 2, \dots, n \qquad (7)$$

Where θ is a vector containing p parameters and $n > p$. We assume further, of course, that f is nonlinear in $\acute{\theta} = [\theta_1, \theta_2, \dots, \theta_p]$. Suppose we call the vector $\widehat{\theta}$ the estimator of θ that minimizes

$$SS_{Res} = \sum_{i=1}^{n}\left[y_i - f\left(x_i, \widehat{\theta}\right)\right]^2 \qquad (8)$$

Suppose we also make the assumptions that of the ε_i are independent and normal with mean zero and varianceσ^2. We know $\widehat{\theta}$ is a maximum likelihood estimator of θ. However, under these circumstances, one cannot make any general statements about the properties of the estimators expect for large samples. In other words, the properties are asymptotic properties. The unbiasedness and minimum variance properties are only approached as the sample size grows large. As a result, for a specific nonlinear model and a specific sample size, nothing can be stated regarding the properties of the estimators. There are asymptotic variance-covariance results that we can be used to obtain approximate confidence intervals and to construct t-statistics on the parameters.

The method most often used in software computing algorithms for finding the least squares estimator $\widehat{\theta}$ in a nonlinear model is the Gauss-Newton procedure (see; Bard (1974), and Bates and Watts (1988)).

We shall denote these estimates by the vector $\widehat{\theta}_0 = \left(\theta_{1,0}, \theta_{2,0}, \dots, \theta_{p,0}\right)$. In the attempt to find the value of θ that minimizes the residual sum of squares in Equation (8), we first

expand the nonlinear function in Equation (7) in a Taylor series around $\theta = \theta_0$ and retain only linear terms. Thus

$$f(x_i,\theta) \cong f(x_i,\theta_0) + \left(\theta_1 - \theta_{1,0}\right)\left[\frac{\partial f(x_i,\theta)}{\partial \theta_1}\right]_{\theta=\theta_0} + \left(\theta_2 - \theta_{2,0}\right)\left[\frac{\partial f(x_i,\theta)}{\partial \theta_2}\right]_{\theta=\theta_0} + \cdots + \left(\theta_p - \theta_{p,0}\right)\left[\frac{\partial f(x_i,\theta)}{\partial \theta_p}\right]_{\theta=\theta_0} \quad i = 1,2,\dots,n \qquad (9)$$

Equation (9) introduces what is essentially a linearization of the nonlinear form $f(x_i,\theta)$ in (7). The reader may view Equation (9) as a linear approximation in a neighborhood of the starting values.

3.2.2 Maximum Likelihood Estimators:

We note that if Jacobian have θ than least squares not maximum likelihood.

The normal equations of Maximum Likelihood are:

$$\frac{\partial \ln L}{\partial B} = \frac{1}{\sigma^2}\sum_{i=1}^{n} \epsilon_i \frac{\partial h\,(x_i,B)}{\partial B} = 0$$

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^{n} \frac{1}{J_i}\left(\frac{\partial J_i}{\partial \theta}\right) - \left(\frac{1}{\sigma^2}\right)\sum_{i=1}^{n} \epsilon_i \frac{\partial g\,(y_i,B)}{\partial \theta}$$

$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n} \epsilon_i^2 = 0$$

For solving these equations, we use Newton Raphson method.

The log likelihood function is

$$\ln L = \frac{-n}{2}\ln 2\pi - \frac{n}{2}\ln \sigma^2 + \sum_{i=1}^{n} \ln J(y_i,\theta) - \frac{\sum_{i=1}^{n}\left[g(y_i,\theta) - h(x_i,B)\right]^2}{2\sigma^2}$$

We note that if Jacobian have θ than least squares is not maximum likelihoode.

The normal equations of maximum likelihood are

$$\frac{\partial lnL}{\partial B} = \frac{1}{\sigma^2}\sum_{i=1}^{n} \epsilon_i \frac{\partial h(x_i,B)}{\partial B} = 0$$

The nonlinear Least Squares Estimator and Maximum likelihood estimation method is consistent, efficient (see;

William H. Greene (2003)).

4. The Empirical Study

4.1 Data of Study

We are use data about Egypt economics cover the period from 2000 to 2017 source of data is https://www.ceicdata.com/en/indicator/egypt/exchange-rate-against-usd . The variables of the study are: Exchange Rate; Inflation; Exports; Imports; Investments; Budget Deficit.

4.2 Study the Relationship Between the Variables

We start with plot between the dependent variable (Y: exchange rate) and the independent variables are (X_1: inflation), (X_2: exports), (X_3: imports), (X_4: investments), (X_5: budget deficit). The resulting scatterplot shows that:

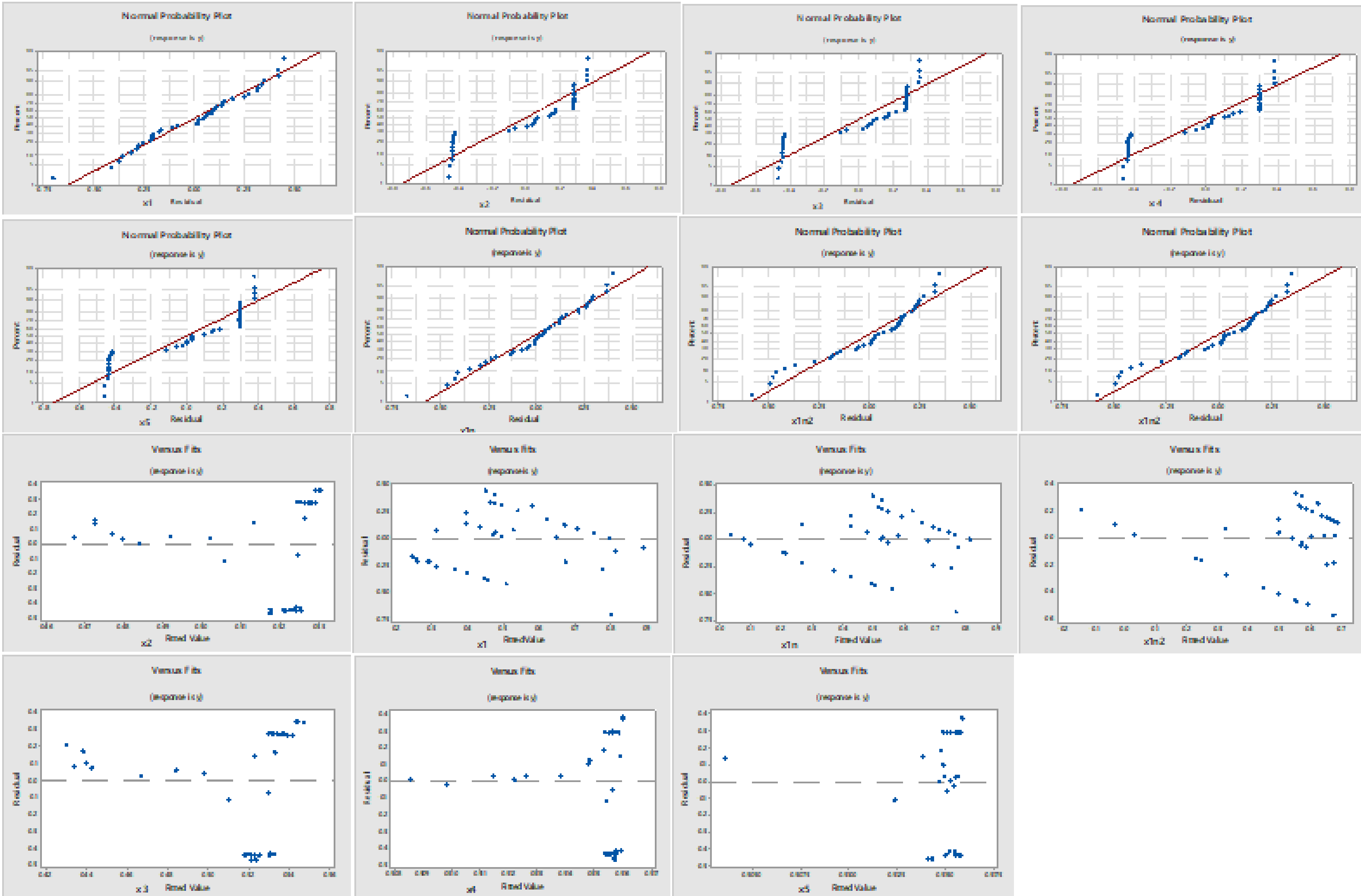

Figure 1: Normal Probability and Residuals Plots for all Dependent Variables and Exchange Rate

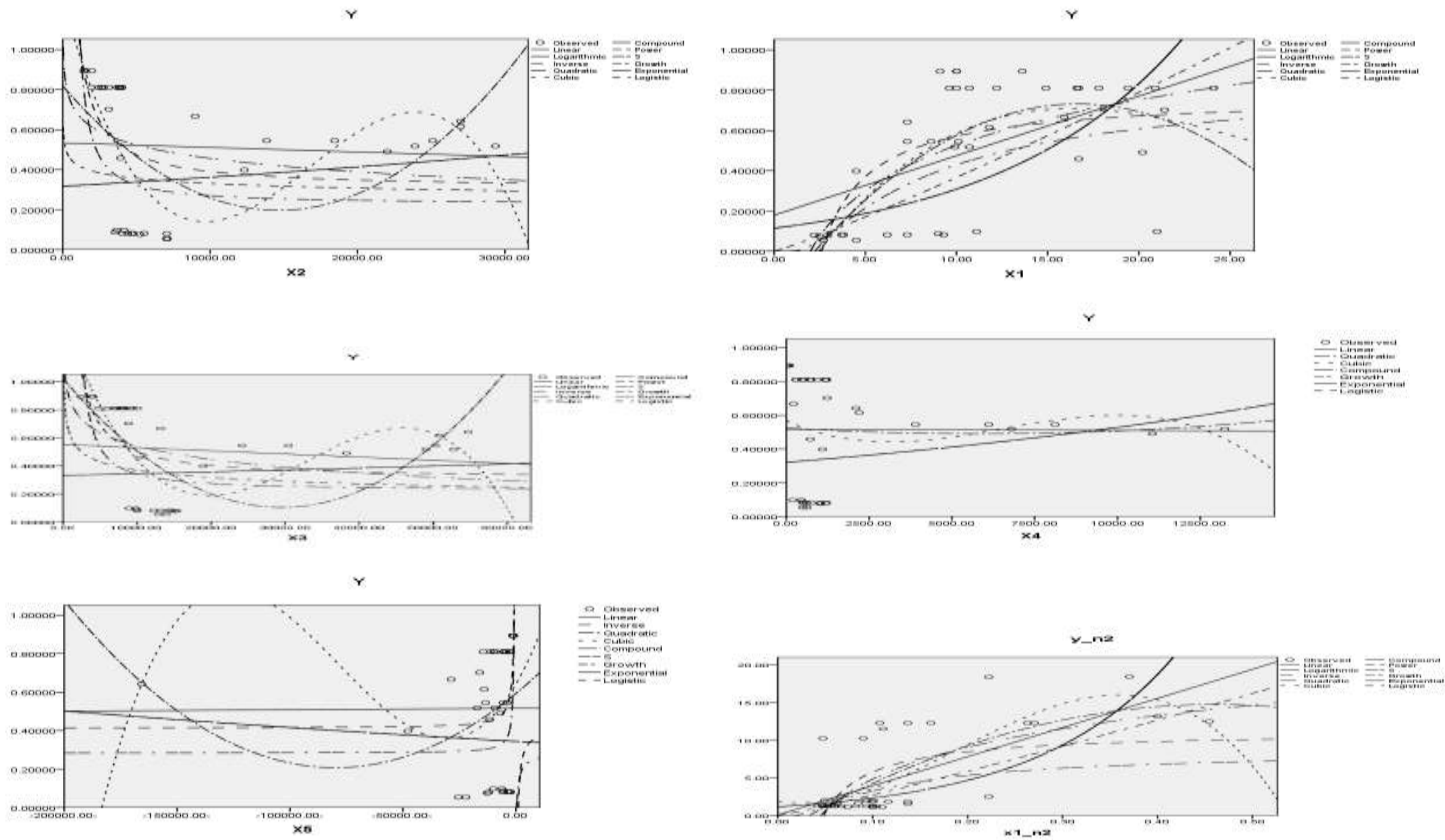

Figure 2: Curve Estimation for all Dependent Variables and Exchange Rate

In the next part several models used and proposed for the data are presented, and the coefficient of determination R^2 was

Table (1): R^2 of models for each variables					
R^2 of models	X_1	X_2	X_3	X_4	X_5
Linear	.293	.003	.011	.000	.000
Logarithmic	0.398	0.069	0.113	.	.
Inverse	0.392	0.213	0.237	0.112	0.112
Quadratic	0.450	0.190	0.341	0.049	0.049
Cubic	0.453	0.396	0.492	0.061	0.061
Compound	0.314	0.011	0.003	0.021	0.002
Power	0.437	0.008	0.027	.	.
S	0.444	0.094	0.116	.	0.051
Growth	0.314	0.011	0.003	0.021	0.002
Exponential	0.314	0.011	0.003	0.021	0.002
Logistic	0.314	0.011	0.003	0.021	0.002

calculated for each model as showed in table (1)

4.3 Estimation Parameters of Nonlinear Models

From the Figure (1), (2) and Table (1), we find that the variable (X_1: inflation) is the most important for analysis and estimation of the dependent variable (Y: exchange rate) but the relation between X_1 and Y is nonlinear so that we will use transformation like inverse for both of them. The next table presents the summary of the model between the inverse of Y as a dependent variable (Y_n2) and the inverse of variable X_1 as an independent variable (X_1_n2).

Table (2): Summary of the Model and Parameters Estimation									
Dependent Variable:y_n2									
	Summary of the Model					Parameters Estimation			
Equation	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Linear	.480	34.162	1	37	.000	.078	38.719		
Logarithmic	.462	31.764	1	37	.000	18.877	6.099		
Inverse	.324	17.764	1	37	.000	11.181	-.541-		
Quadratic	.506	18.467	2	36	.000	-2.394-	74.570	-81.066-	
Cubic	.534	13.358	3	35	.000	1.874	-18.103-	428.468	-7.470E2
Compound	.444	29.555	1	37	.000	1.163	1.045E3		
Power	.437	28.717	1	37	.000	34.944	1.107		
S	.314	16.931	1	37	.000	2.169	-.099-		
Growth	.444	29.555	1	37	.000	.151	6.951		
Exponential	.444	29.555	1	37	.000	1.163	6.951		
Logistic	.444	29.555	1	37	.000	.860	.001		
The independent variable is x_1_n2.									

From the Table (2), we result that the relation between X_1, Y still nonlinear and cubic form. For linear the R square is enhancing from 0.29 to 0.48 and cubic the R square is enhancing from 0.45 to 0.53.
The Johnson Schumacher nonlinear regression model is used for obtaining the outstanding model for estimating of exchange rate.
Johnson Schumacher Model:

$$Y = b_1 * exp\left({-b_2}/{(x + b_3)} \right)$$

Table (3):Parameters Estimation of Johnson Schumacher Model				
Parameters	Estimate	Standard Error	95% of Confidence Interval	
			L. Bound	U. Bound
b1	.477	.006	.464	.489
b2	-.398-	.018	-.435-	-.362-
b3	-.998-	.004	-1.005-	-.991-

From the Table (3), we result that all Johnson Schumacher model's parameters are significant. The estimated Johnson Schumacher model is:

$$Y = 0.477 * exp\left({0.398}/{(x - 0.998)} \right)$$

The value of R^2 is 0.994 means that the model accounts about 99.4% of variability in dependent variable. The result shows that the Johnson Schumacher model is significant. R^2 = 1 - (Sum Squares of Error) / (Corrected Sum Squares) = 0.994.

5. Conclusion and Remark:

This study presents the following variables: (X_1: inflation), (X_2: exports), (X_3: imports), (X_4: investments), (X_5: budget deficit). These represent the main variables that affect (Y: exchange rate). The study introduced nonlinear models and focus on two nonlinear regression models; cubic and Johnson Schumacher nonlinear model. The Johnson Schumacher nonlinear model shows outstanding performance.

Reference

Adamowski, J. Chen,H. F., Prasher, S. O., Zielinski, B. O., Sliusarieval, A. (2012) Comparison of multiple linear and nonlinear regression, autoregressive integrated moving average, artificial neural network, and wavelet artificial neural network methods, Water Resources Research, Vol. 48, 1-12.

Archontoulis, S.V., Miguez ,F.E., and Moore, K.J., (2014). Evaluating APSIM Maize, Soil Water, Soil Nitrogen, Manure, and Soil Temperature Modules in the Midwestern United States. Agronomy Journal 106 (3):1025-1040.

Archontoulis, S. V. and Miguez, F.E. (2015) Nonlinear Regression Models and Applications in Agricultural Research, Agronomy Journal, Volume 107, Issue 2, 786- 798.

Bard, Y. (1974) Nonlinear Parameter Estimation. New York: Academic Press.

Bates, D.M., and Watts., D.G. (1988). Nonlinear Regression Analysis and Its Applications. New York: John Wiley & Sons.

Chen, M., (2010), A Comparison of Nonlinear and Nonparametric Regression Methods, partial fulllment of the requirements for the degree of Master of Applied Statistics. Department of Mathematics and Statistics, Memorial University of Newfoundland, March 12, 2010.

Chiyenda, S. S., & Kozak, A. (1984). Additivity of component biomass regression equations when the underlying model is linear. Canadian Journal of Forest Research, 14(3), 441-446.

David,,I.,J, Asiribo,O.E., and Dikko,H.G. (2019) Parameter Estimation of Nonlinear Split-Plot Design Models: A Theoretical Framework, Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online): 2229-5666 Vol. 12, Issue 1 (2019): 117-129.

Huang, X., Tang, X., Deng ,X. and Wang,X. (2016), The large deviation for the least squares estimator of nonlinear regression model based on widely or than t dependent random variables WOD errors, Journal of Inequalities and Applications, Vol. (125), 1-11.

Johnson N., (1935). A trend line for growth series. J Am Stat Assoc 30(192):717

Kaneko, H., & Funatsu, K. (2013). Classification of the degradation of soft sensor models and discussion on adaptive models. AIChE Journal, 59(7), 2339-2347.

Khan, I. U., Sajid, S., Javed, A., Sajid, S., & Shah, S. U. (2017). Comparative diagnosis of typhoid fever by polymerase chain

reaction and widal test in Southern Districts (Bannu, Lakki Marwat and DI Khan) of Khyber Pakhtunkhwa, Pakistan. Acta Sci. Malaysia, 1(2), 12-15.

Kimura, K., & Waki, H. (2018). Minimization of Akaike's information criterion in linear regression analysis via mixed integer nonlinear program. Optimization Methods and Software, 33(3), 633-649.

Limaa, D.C., Limaa,G.B.A., Quelhas, O. L. G., & Ferreirab, R. N. (2015). Integrated operations: value and approach in the oil industry. Brazilian Journal of Operations & Production Management, 12(1), 74-87.

Mahaboob, B,.Venkateswarlu, B., Mokeshrayalu, G. and Balasiddamuni,P., (2017), A different approach to estimate nonlinear regression model using numerical methods, IOP Conf. Series: Materials Science and Engineering, 263, 1-7.

Motulsky, M.J. and Brown, R. E. (2006) Detecting outliers when fitting data with nonlinear regression, BMC Bioinformatics, 7:123, 1-20

Özsoy, V. S., & Örkçü, H. H. (2016). Estimating the parameters of nonlinear regression models through particle swarm optimization. Gazi University Journal of Science, 29(1), 187-199.

Payandehl, B., (1983). Some Applications of Nonlinear Regression Models in Forestry Research. The Forestry Chronicle Downloaded from pubs.cif-ifc.org by 41.68.223.196 on 09/12/17

Sukumar, N., (2017). Economics of the international financial system. New Delhi: Routledge.
P. 173. ISBN 9781317342236. OCLC 927438010

Schumacher F., (1939) A new growth curve and its relation to timber yield studies. J Forest 37(1):819–820

Tang, N.S., and Zhao,P,U (2013) Empirical likelihood-based inference in nonlinear regression models with missing responses at random, Statistics, Vol. 47, No. 6, 1141–1159, http://dx.doi.org/10.1080/02331888.2012.658807

Tian,C., and Hao, Y., (2018) A Novel Nonlinear Combined Forecasting System for Short-Term Load Forecasting: Chengshi Tian and Yan Hao www.mdpi.com/journal/energies, 1-34.

William H. Greene (2003). Econometric Analysis, fifth edition, New York University, prentice Hall. International, Inc. Prentice Hall, Upper Saddle River, New Jersey 07458.

Xiao, X., White,P.E, Hooten, M. B and Durham,S.L. (2011) On the use of log-transformation vs. nonlinear regression, Ecology, 92(10), 1887–1894.